General Equilibrium Effects of Student Loans on the Provision and Demand for Higher Education*

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Abstract

We characterize the outcomes of the tertiary education market in a context where borrowing constraints bind, there is a two-tier college system operating under monopolistic competition in which colleges differ by the quality offered and returns to education depend on the quality of the school attended. Our main finding shows that subsidized student loan policies can lead to a widening gap in the quality of services provided by higher education institutions. This happens because the demand for elite institutions unambiguously increases when individuals can borrow. This does not necessarily happen in non-elite institutions, since relaxing borrowing constraints makes some individuals move from non-elite to elite institutions. To the extent that the increase in demand for elite institutions is larger than non-elite, the former can increase prices, cut-offs and investment per student more than the latter. Our results show that, when analyzed in a general equilibrium setting, subsidized loan policies can have second order effects that affect market shares.

JEL Classification: E24, I23, I24, I25, I28,L130

Keywords: Higher education, Student Loans, Education policy, Returns to Education, Labor productivity, Monopolistic Competition

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1 Introduction

The market for higher education has received significant attention in the economics literature. In particular, the effects that subsidized loan policies have on the demand side of the market have been widely studied, given the dramatic increase in student debt during the last two decades in the U.S. Overall, there is a consensus in the literature on the fact that credit constraints explain only a small fraction of enrollment decisions in higher education in the U.S. However, this is not the case in developing countries, where student financial aid systems are weak and evidence suggests that college enrollment is highly determined by family wealth (World Bank, 2003, 2012).[1] Within this context, the implementation of subsidized student loan policies can potentially increase the demand for education, which may have additional equilibrium effects, such as increases in tuition prices and changes in the quality of services offered by colleges.

Understanding the effects of subsidized student loan policies is of central importance, given the massive investments that have been made in student credit programs during the last two decades in the developing world, Latin America and some African countries[2]. The demand side effects of these policies in a context where borrowing constraints determine enrollment decisions have been studied by the literature and the conclusions are certainly appealing: an expansion in student loans increases the demand for higher education among the most able students, which reduces the inefficiency that exists when very high-ability individuals with low initial wealth cannot access tertiary education (Canton and Blom, 2004). This partial equilibrium analysis unambiguously suggests that such policies have welfare improving effects on its beneficiaries. As a consequence, these programs have often received the support of international organizations, such as the World Bank and the Inter-American Development Bank[3].

However, the implementation of subsidized student loan programs has general equilibrium effects that have not been deeply studied by the literature. The increase in demand for education generated by student loan policies can potentially affect tuition prices and quality offered by

[1] In this paper we use the terms "college" and "universities" indistinctly.

[2] See World Bank (2005) for a review on student loan programs in Latin American and African countries.

[3] These institutions have contributed to different student loan projects in the developing world. For example, the World Bank has been financing the Colombian ACCES program since 2002 and committed in 2014 to lend $200 million during the period 2014-2019. Recently, the IDB provided a $10 million dollar loan to the Higher Education Finance Fund in 2012, to finance student loan programs in 4 Latin American countries.

certain colleges. This general equilibrium effects can affect the welfare of individuals that do not have access to the loan programs, which might offset the overall benefits of the policies.[4]

This paper contributes to the literature that studies the consequences of subsidized loan policies by analyzing the general equilibrium effects that such programs have on the quality of education provided by different tiers of colleges. We assume there are two tiers of colleges, which we denote as low and high-quality, or elite and non-elite colleges, that operate under monopolistic competition (Dennis Epple and Sieg, 2006). Colleges choose the skills threshold for admission, tuition rates, and investments per student to maximize the quality of the education offered, which is a function of the skills of the student body and total investments per student. In equilibrium, students who attend elite colleges have higher expected returns in the labor market, when compared to students who attend the non-elite system. Individuals choose whether and which college to attend, given the expected returns of each option. We characterize the demand for higher education, the incentives of each tier to invest and admit students, and the equilibrium consequences of subsidized-loan policies.

Our model rationalizes how a widening of the education quality gap can arise as a consequence of a subsidized student loan policy. We find a set of equilibria in which subsidized student loans widen the gap of the quality supplied by elite and non-elite institutions. When the loan program is implemented, the demand for elite colleges unambiguously increases, as the loans loosen the borrowing constraints of high-skilled individuals with low wealth. Higher demand allows elite universities to increase the skill acceptance threshold and, as a consequence, the average skills of the student body, while maintaining budget balancedness. As long as the skills of the student body and investments per student are complements in the production function for education quality, colleges have incentives to increase tuition and investments per student, generating a further increase in quality. In contrast, the demand for non-elite colleges does not necessarily increase with the student loan policy. Even though some individuals that would not study in the absence of the program will enroll in non-elite schools, some others that would attend non-elite

[4] Obiols-Homs (2011) argues that in an incomplete markets setting, although increasing borrowing limits increases the welfare of borrowing constrained individuals, in equilibrium this also leads to an increase in the interest rate paid by the borrowers. The two effects oppose each other, so the effect of loosening borrowing limits on welfare is ambiguous and follows a U-shape. Although we do not take into account the effect of borrowing constraints on the interest rate and assume government student loans are subject to an exogenous interest rate, his findings strengthen our theory that student loan policies might have negative effects on welfare, in equilibrium.

institutions in the absence of the program will decide to attend elite schools when they have access to loans. The effect of the subsidy program on the average skills of the student body of non-elite colleges does not necessarily increase, nor do investments per student. This generates a widening in the gap of the education quality provided.

The results presented give us tools to discuss the design of the optimal student loan policy in a context where the government has outside funds that has to allocate within the existing population. There are two opposing forces in a student loan program. On one hand, the student loan policy that gives priority to the lowest-ability individuals that are borrowing constrained has the largest impact on enrollment, given that marginal benefits of education are larger for those households, who have lower lifetime expected income. On the other hand, given that education quality depends on the ability of the student body, there is an incentive to offer subsidized student loans to high-ability individuals. This increases the quality of education, generating a positive externality on every other student attending those colleges. The trade-off of a student loan policy is increasing enrollment at the expense of lower quality.

Our analysis is novel given that we focus on developing countries, as opposed to the structural literature that has only explored the U.S. context, as far as our knowledge goes. The educational sector in developing economies is particularly different from that of developed economies, for three main reasons. First, there is evidence that credit constraints play a role in determining college enrollment decisions among households of developing countries (Melguizo et al., 2015), as opposed to the case of developed countries. Second, in many developing countries, private institutions own a larger share of the market for higher education, as compared to European countries or even the U.S. (see Figure 1). This is important because public institutions may not be as responsive to market incentives, but rather follow the social planner's objectives. In contrast, private institutions are potentially more responsive to market signals, so any change in demand will generate stronger equilibrium effects in developing economies. Third, enrollment rates in developing countries are very low, when compared to enrollment in developed countries. As documented by Mestieri (2016), there is an existing positive correlation between enrollment rates and income per capita at a cross-country level.

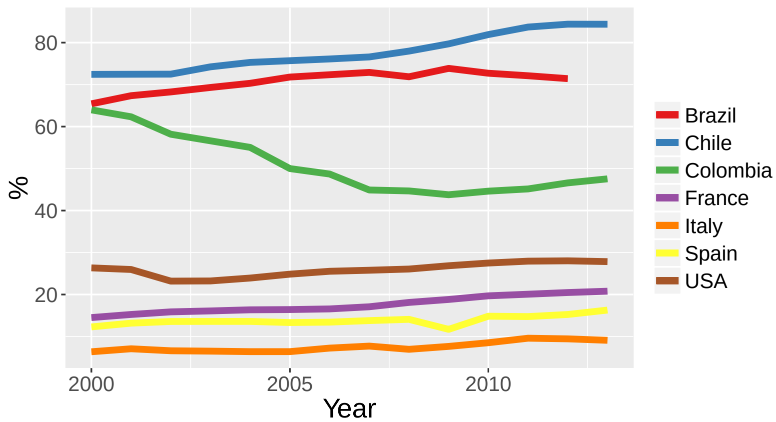


Figure 1: % of enrollment in private institutions by country.

Our motivation is the case of Colombia, a developing country that underwent a massive expansion of publicly supplied student loan availability during the last decade. After the introduction of the policy, the number of students enrolled increased. However, there has been a widening gap in the quality offered by elite and non-elite universities, measured as average test scores in entry and exit examination tests, the number of professors per students and various measures of academic production such as articles published per faculty.

The rest of the paper is organized as follows. In Section 2 we describe the relevant literature. In Section 3 we describe a model of the market for higher education, characterize the demand for a two-tiered education system and explains the mechanism through which borrowing constraints affect equilibrium quality supplied. In this section, we illustrate the main theoretical results of the paper. That is, a policy leading to subsidized loans can increase the gap of quality of education. We describe the case and Colombia and illustrate how this case is consistent with what we predict in the theoretical model in Section 4. In Section 5 we describe the numerical analysis and calibration exercise. We conclude in Section 6 concludes.

2 Related Literature

This paper is related to various lines of the literature on the economics of education. First, our paper is related to the literature studying the relevance of borrowing constraints in the access to higher education. Given that we are studying the welfare effects of government loan policies in developing countries, knowing whether borrowing constraints matter is of central importance. Although there is evidence suggesting that borrowing constraints do not determine school attendance of students in advanced economies (Carneiro and Heckman, 2002; Keane and Wolpin, 2001), the existing evidence in developing countries highlights the importance of borrowing constraints in the educational decisions (Attanasio and Kaufmann, 2009; Kaufmann, 2014; Melguizo et al., 2015).

Second, our paper is related to the literature that studies general equilibria in the market for education. This literature has mostly studied what is known as the *"Bennett Hypothesis"*, which states that an expansion in the availability of funding for students is almost totally appropriated by colleges through increases in tuition prices. As the former U.S. Secretary of Education stated in 1987, *"If anything, increases in financial aid in recent years have enabled colleges and universities blithely to raise their tuitions, confident that Federal loan subsidies would help cushion the increase"*[5]. There is mixed evidence on the effects of student loan policies on tuition prices, depending on the nature of the universities and funding provided (Singell and Stone, 2007; Rizzo and Ehrenberg, 2002; Gordon and Hedlund, 2015). Our paper adds to this literature in two dimensions. First, we study equilibrium effects that go beyond prices, as we analyze the effects on the quality offered by colleges. Second, this literature has been focused extensively in the United States. In this paper, we provide a new context of analysis for general equilibrium effects in the market for higher education: a developing economy where credit constraints bind.

Finally, our paper is related to the industrial organization literature analyzing the behavior of colleges in non-competitive markets. We implement the framework used by Dennis Epple and Sieg (2006), as we model the supply side of the educational sector as an oligopoly sector in which a fixed amount of colleges interact to attract students and maximize the quality of the education they offer, subject to a balanced budget constraint. Quality by universities is a composite of average student ability, to resemble peer effects in schooling, and the average investments per student. This treatment of quality has been standard in the literature that models schools (Caucutt, 2001). We extend the model to assess the general equilibrum effects of tuition prices, college quality, and admittance rules, in the context of a developing economy. We treat wages of college graduates

[5] William Bennett to the New York Times, 1987.

as a function of the quality supplied by the school attended. To the best of our knowledge, this approach has not been used in the structural literature, but there is empirical evidence that relates future wages to the quality of the education (Black and Smith, 2006; Dan Black and Daniel, 2005; Zimmerman, 2014).

3 A Model of the Market for Higher Education with Credit Constraints

There are two types of agents in the economy: households and universities. There is a government that offers educational credits to high-ability individuals that decide to attend college, at an exogenous interest rate $R \geq r$, where r is the risk free interest rate. In addition, the government subsidizes the interest paid by the poorest households that access the credit, at a subsidy rate s. In order to finance these subsidies, the government levies a marginal tax, τ, to every household in the economy. The government policies are exogenous, fixed before the economy starts and satisfy budget balance. Given these policies, the market of higher education operates under monopolistic competition. Universities supply human capital in the market for education, by choosing a tuition price, a minimum ability level for admission and a level of investment per student. Given government and university policies, the households decide if they want to study in any university at the prevailing market prices.

3.1 Households

Households are born with innate ability and wealth (θ, b), according to a bivariate distribution $F(\theta, b)$ over the space $[0, 1] \times [\underline{b}, \bar{b}]$. Individuals live for two periods, after which they die with probability equal to one. In period 1, individuals choose either to study at the university or work in the non-skilled labor market at a wage w per efficiency unit of labor. Individuals that do not study receive a wage θw, do not have access to credit markets and can save at the risk-free rate r. There are two universities in the economy denoted by h and l. Each university sets a threshold $\underline{\theta}^j$ for $j = h, l$ such that only students that have ability $\theta \geq \underline{\theta}^j$ are admitted to university j, and we assume that this information is public[6]. Therefore, individuals with $\theta < \min\{\underline{\theta}^h, \underline{\theta}^l\}$ cannot study and have to work. Individuals who decide to study at university j cannot work, and have to pay a tuition, P^j, set by the university.

In order to finance education, the government offers student loans of up to the price of the tuition, P^j, at the interest rate R to people that decide to study and have an ability level $\theta \geq \theta_{min}$. In addition, students with low wealth, $b \leq b_{max}$, that decide to study and have access to the loan will receive a subsidy on the interest rate, s. Loans are given conditional on studying, and individuals that study and are eligible for the loan choose whether to borrow from the government or not. In order to finance these subsidies, the government levies a proportional tax, τ, to every individual in the economy. Individuals for which $\theta < \theta_{min}$ are borrowing constrained and can only finance education with their initial wealth. Therefore, in the first period the household decides its level of consumption, c, whether to study or not in any university, h, l, and the level of savings, a, which can be potentially negative for households that study and satisfy the government conditions for the educational loans.

In the second period, the households are either non-, low- or high-skilled, depending on whether they decided to study in the first period and which college they attended. Those who decided to study in period 1, will enter the j-skilled labor market in period 2, and receive a wage equal to $w\theta(1+z^j)$, where z^j is a skill premium that is university specific. This quality is an equilibrium object that depends on the quality of the student body and investments per student, and is fully characterized in the next section. We assume that individuals have perfect foresight of the value of z^j for $j = h, l$ when they optimize. Individuals who do not study will become part of the non-skilled labor force at a wage $w\theta$. We exclude the possibility of default in the model by assuming that repayment is fully enforced, so in the second period individuals that have government debt will repay their student loan. Given prices R, r, w, government policies τ, s,

[6] We assume that $\underline{\theta}^j, j = h, l$ is a public threshold, since our purpose is not to study the frictions in the college application process, as opposed to some papers in the literature that model explicitly these information frictions (Hector Chade and Smith, 2014; Fu, 2014).

university policies $\{\underline{\theta}^j, P^j\}_{j=h,l}$, and perfect foresight about education quality $\{z^l, z^h\}$, a household that is eligible for studying at the university j, $\theta \geq \underline{\theta}^j$, and decides to study gets a utility equal to:

$$\begin{aligned} V^j(\theta, b) = \max_{c,a} \quad & u(c) + \beta u(c'), \quad \text{s.t.} \\ & c + a + P^j = b \\ & c' = a(1+r) + w\theta(1+z^j) \\ & a \geq \bar{A}, \quad c \geq 0, \quad c' \geq 0 \end{aligned} \tag{1}$$

Individuals that decide not to study, get the following utility:

$$\begin{aligned} V^N(\theta, b) = \max_{c,a} \quad & u(c) + \beta u(c'), \quad \text{s.t.} \\ & c + a = b \cdot (1 - \tau) + w\theta \\ & c' = a(1+r) + w\theta \\ & a \geq 0, \quad c \geq 0, \quad c' \geq 0 \end{aligned} \tag{2}$$

The individual with ability and wealth (θ, b) decides to study at university j whenever $\theta \geq \underline{\theta}^j$ and $V^j(\theta, b) \geq V^N(\theta, b)$, and $V^j(\theta, b) \geq V^{-j}(\theta, b)$ if they can attend to the other university $-j$, i.e. $\theta \geq \underline{\theta}^{-j}$. Otherwise, the individual decides not to study. Therefore, the household's value function is given by:

$$V(\theta, b) = \begin{cases} \max\{V^h(\theta, b), V^l(\theta, b), V^N(\theta, b)\} \text{ if } \theta \geq \max\{\underline{\theta}^h, \underline{\theta}^l\} \\ \max\{V^j(\theta, b), V^N(\theta, b)\} \text{ if } \underline{\theta}^{-j} > \theta \geq \underline{\theta}^j \\ V^N(\theta, b) \text{ if } \theta < \min\{\underline{\theta}^h, \underline{\theta}^l\} \end{cases}$$

The following section gives a detailed characterization of the demand for both tiers of schools in the state space. This characterization will allow us to give insights on the optimal student loan policy on a monopolistically competitive market.

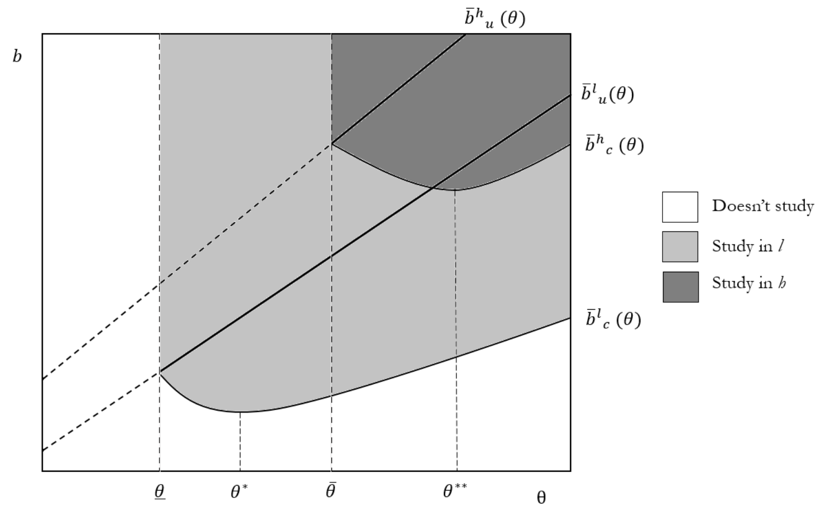


Figure 2: Representation of the education decisions on the state space.

3.1.1 Characterization of the Demand

For a given set of initial parameters, the shaded region in Figure 2 illustrates the individuals that choose to study in the state space when both universities set their acceptance threshold to 0 and there are no government-supplied student loans. The following sequence of theorems characterize the demand for college education on the state space, and its close relationship with borrowing constraints. This will let us derive some results about the socially optimal student loan policy. First, we describe the college decision for households that are unconstrained.

Theorem 1. *Among the unconstrained households, the decision of whether and where to study is independent of initial wealth, b, and follows a cut-off rule on θ. That is, there exist $\underline{\theta}$ and $\bar{\theta}$ such that:*

- *If $\theta \leq \underline{\theta}$, the individual will not study.*
- *If $\underline{\theta} \leq \theta \leq \bar{\theta}$, the individual will attend the low-quality college.*
- *If $\bar{\theta} \leq \theta$, the individual will attend the high-quality college.*

where:

$$\bar{\theta}_l = \frac{1+r}{w}\left(\frac{P_l}{z_l-(1+r)}\right), \quad \bar{\theta}_h = \frac{1+r}{w}\left(\frac{P_h-P_l}{z_h-z_l}\right)$$

Proof. See Proof A.1 □

Theorem 1 is a result of the fact that ability θ, unskilled labor w and quality of the school attended z_j are complements. In particular, this complementarity implies: *a*) among the unconstrained individuals, those with higher ability face higher marginal returns of education, so will choose, *ceteris paribus*, a higher quality school for a given wealth, *b*) as the wages of unskilled labor w increase, the marginal returns to education rise for every θ, so marginal individuals will shift to higher levels of education, *c*) if college j, for $j \in \{l, h\}$, increases its price P_j or reduces its quality z_j, marginal individuals will change their schooling decision in the expected direction. That is, if P_j increases or z_j decreases, marginal individuals will change their decision of attending school j. Finally, *d*) if the interest rate r increases, present consumption becomes more valuable than future consumption, so marginal individuals will reduce their present expenditures in education. Theorem 2 characterizes the individuals that, given their decision to attend college j, are borrowing constrained.

Theorem 2. *Given an ability θ, there exist cut-offs, $\bar{b}^j_u(\theta)$, $j \in \{N, l, h\}$, on the initial wealth, such that individuals with $b \geq \bar{b}^j_u(\theta)$ that attend college j will not be borrowing constrained. Individuals that attend college j and have $b < \bar{b}^j(\theta)$ will be borrowing constrained and will not be able to smooth consumption over time. The cut-offs are linear, increasing in θ and take the form:*

$$\bar{b^N_u}(\theta) = -\bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1 - (\beta(1+r))^{-1/\sigma})$$

$$\bar{b^l_u}(\theta) = P_l + (\beta(1+r))^{-1/\sigma} w\theta(1+z_l) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

$$\bar{b^h_u}(\theta) = P_h + (\beta(1+r))^{-1/\sigma} w\theta(1+z_h) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

Proof. See Proof A.2 □

Given a level of education and initial wealth, individuals with a higher θ have higher lifetime income and in an unconstrained world would consume more in every period of their lives. Given the existence of a borrowing limit $\bar{A}$, for a sufficiently high θ individuals will be borrowing constrained. As a consequence, the initial wealth that individuals must have not to be borrowing constrained is increasing in ability. Figure 2 illustrates the cut-off functions $\bar{b}^j_u(\theta)$ on the state space. As illustrated, individuals above the $\bar{b}^j_u(\theta)$ function, will decide to study in college j whenever her θ falls inside the corresponding interval in the cut-offs defined in Theorem 1. Note also that individuals that are borrowing constrained when studying at college l will also be borrowing constrained when studying in h, assuming a higher price of education in the high-quality college (which, of course, is an equilibrium object). Moreover, the functions $\bar{b}^j_u(\theta)$ are steeper when the quality z_j increases, since quality of schooling and ability are complements. Finally, we do not consider the case in which individuals are borrowing constrained when they do not study. Since in our context, individuals that do not study earn the same wage in every period, they will only be borrowing constrained when the interest rate $\beta(1+r) << 1$. However, for a reasonable calibration, individuals will be able to smooth consumption. The following two theorems illustrate the study decision of individuals that are borrowing constrained.

Theorem 3. *Given ability θ, the decision to study in the low-quality college, l, or not study at all, follows a cut-off strategy on b, such that individuals with $b \geq \bar{b}^l_c(\theta)$ will attend college l, and those with $b < \bar{b}^l_c(\theta)$ will not study. The cut-off is characterized implicitly by equation (17) in the proof. Moreover, if the intertemporal elasiticity of substitution is lower than 1 the cutoff is U-shaped and there exists a θ^* such that $\bar{b}^l_c(\theta)$ is:*

- *decreasing in θ for $\theta \leq \theta^*$*
- *increasing in θ for $\theta \geq \theta^*$*

where θ^* solves:

$$\left(\frac{1}{1-\sigma}\right)(b(\theta^*) - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta^*(1+z_l) - (1+r)\bar{A})^{1-\sigma} -$$

$$\Phi(w\theta^*(2+r) + b(\theta^*)(1+r))^{1-\sigma} = 0$$

$$b(\theta) = \theta\left[\frac{wX(1+z_l) - w(2+r)}{1+r}\right] - X\bar{A}$$

$$X = \left[\frac{\Phi(1-\sigma)(2+r)}{\beta(1+z_l)}\right]^{1/\sigma}$$

Proof. See Proof A.3. □

The cut-off $\bar{b}_c^l(\theta)$ is illustrated in Figure 2, where we assume that the utility function is CRRA with $\sigma = 2$, as is common in the literature, so the intertemporal elasticity is lower than 1. The individuals who are constrained (below $\bar{b}_u^l(\theta)$) will choose either to study at l or not, if their initial wealth exceeds $\bar{b}_c^l(\theta)$. The cut-off is U-shaped because two effects are in action. First, the "complementarity" effect means that, given a b, individuals with higher θ will have higher marginal returns from studying, so are willing to study even though they will not be able to smooth consumption. Therefore, the cut-off is initially decreasing. However, the "constrainedness" effect dominates after some point: given an initial wealth b, individuals with higher θ will face a larger wedge in their Euler equation, meaning that they will be able to smooth consumption to a lower extent. When the wedge is large enough, individuals will prefer not to study and smooth consumption by deciding not to study. Of course, this results strongly depends on the value of σ chosen, and continues to hold for any $\sigma > 1$. For the sake of exposition, in Appendix A.7 we characterize the demand for education with a linear utility function (that is, when $\sigma = 0$ and there is an infinite elasticity of substitution). Figure 15 illustrates the decision of individuals in the state space. As can be expected, in the linear case individuals derive no utility from consumption smoothing, so there does not exist such a "constrainedness" effect. In this case, the threshold is never increasing. The next theorem characterizes the cut-off for individuals that are constrained when studying at h. The results are parallel to Theorem 3.

Theorem 4. *Given ability θ, the decision to study in h or l, follows a cut-off strategy on b, such that individuals with $b \geq \bar{b}_c^h(\theta)$ will attend college h, and those with $b < \bar{b}_c^h(\theta)$ will attend l. The cut-off is characterized implicitly by equation (16) in the proof. Moreover, if the intertemporal elasiticity of substitution is lower than 1 the cutoff is U-shaped and there exists a θ^{**} such that $\bar{b}_c^h(\theta)$ is:*

- *decreasing in* θ *for* $\theta \leq \theta^{**}$
- *increasing in* θ *for* $\theta \geq \theta^{**}$

where θ^{**} *solves:*

$$\left(\frac{1}{1-\sigma}\right)\left(b^*(\theta^{**}) - P_h + \bar{A}\right)^{(1-\sigma)} + \left(\frac{\beta}{1-\sigma}\right)\left(w\theta^{**}(1+z^h) - (1+r)\bar{A}\right) -$$

$$\Phi \times \left(w\theta^{**}(1+z_l) + b(1+r) - P_l(1+r)\right) = 0$$

$$b^*(\theta) = \theta w\left(X^*(1+z^h) - (1+z_l)\right) - X^* + P_l$$

$$X^* = \left(\frac{\Phi \times (1-\sigma)(1+z_l)}{\beta(1+z_h)}\right)^{1/\sigma}$$

Proof. See Proof A.4. □

Having characterized the demand for education in the state space, we can say a couple of things about the relationship between borrowing constraints and the demand. The following result describes the differential effect of relaxing the borrowing limits to households, $\bar{A}$.

Theorem 5 (Borrowing constraints)**.** *If the intertemporal elasticity of substitution is lower than 1, for any given* θ *the cut-offs* $\bar{b}^l_c(\theta)$ *and* $\bar{b}^h_c(\theta)$ *are decreasing on* $\bar{A}$*, and:*

$$\frac{\partial \bar{b}^h_c(\theta)}{\partial \bar{A}} < \frac{\partial \bar{b}^l_c(\theta)}{\partial \bar{A}} < 0 \qquad (3)$$

Moreover, the elasticities of $\bar{b}^l_c(\theta)$ *and* $\bar{b}^h_c(\theta)$ *with respect to the borrowing limit* $\bar{A}$ *are decreasing on* θ*:*

$$\frac{\partial^2 \bar{b}^j_c(\theta)}{\partial \bar{A}\partial\theta} > 0, \quad j \in \{l, h\} \qquad (4)$$

Proof. See Proof A.5. □

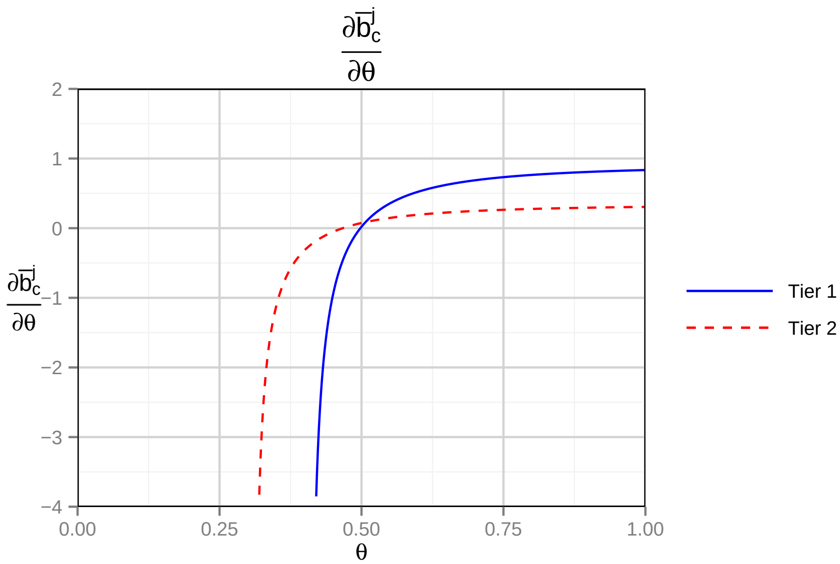


Figure 3: Derivative of $\bar{b}_c^j$ with respect to θ for Tier 1 and Tier 2.

The first part of Theorem 5, given by equation (3), states that, for a given level of ability θ, relaxing the borrowing constraints has a larger impact on enrollment to elite schools, as compared to non-elite schools. That is, relaxing the borrowing constraints by the same amount generates a larger increase in enrollment to elite schools. This is illustrated in Figure 4. The second part of Theorem 5, given by equation (4), states that among the constrained individuals, those with lower θ are more sensitive to relaxing the borrowing constraints. That is, if the borrowing constraints were relaxed by the same amount to all the individuals, more low-θ individuals would change their study decision. This result is a consequence of the decreasing marginal utility. Individuals with high θ and sufficiently low initial wealth have a trade-off between earning relatively high wages in every period and smoothing consumption it they do not study, or studying to earn large wages in the second period at the expense of a very low consumption in the first period. However, because of decreasing marginal utility, the utility of a very large wage in the second period is not as large as for lower θ individuals, so individuals will optimally decide to study only when there is a large increase in the borrowing limits of the first period.

This theorem shows that, after increasing the borrowing limit, the entering cohort of elite schools will be composed, on average, of individuals with higher ability. This result comes through

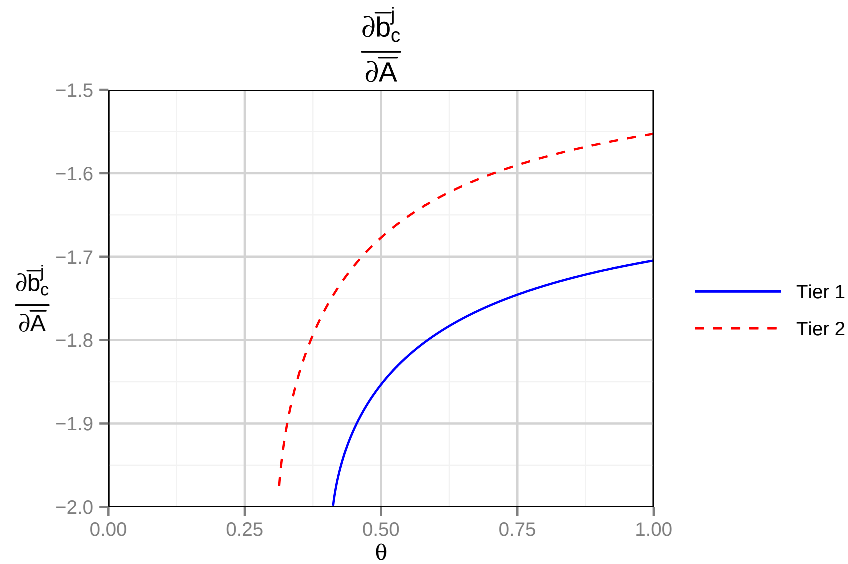


Figure 4: Derivative of $\bar{b}_c^j$ with respect to the borrowing limit $\bar{A}$ for Tier 1 ($\frac{\partial \bar{b}_c^h}{\partial \bar{A}}$) and Tier 2 ($\frac{\partial \bar{b}_c^l}{\partial \bar{A}}$).

two channels. First, for any given level of ability, the number of students that will attend the elite school is larger. This means that for every student of certain ability that enters the non-elite school, more than one student will move from the non-elite to the elite school. The second effect is that relaxing borrowing limits has a stronger effect on lower ability individuals, which implies that any student loan program will increase the

on the design of an optimal student loan policy in a partial equilibrium setting. If the objective of the government is to maximize enrollment, the policy should target the lower ability individuals. As a matter of illustration of Theorem 5, Figure 5 illustrates the number of individuals of ability θ that change their study decision as the borrowing constraint is relaxed from $\bar{A} = 0$. As stated in Theorem 5, the individuals in the state space with low ability that would study in the unconstrained world (those with $\theta \in [\bar{\theta}_l, \bar{\theta}_h]$) are more sensitive to relaxing borrowing constraints. Therefore, increasing the borrowing capacity increases enrollment more among the low ability individuals.

3.2 Universities

Universities act as firms that maximize an objective function. Given that university systems in most countries are non-profit firms, we follow the literature on education and industrial organization

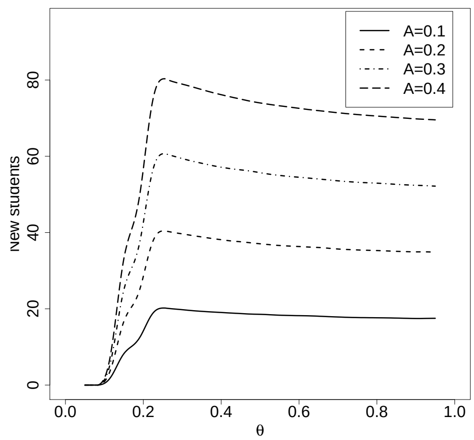


Figure 5: Number of students that change their study decision when borrowing constraints change from $\bar{A} = 0$ to $\bar{A}$, by ability θ.

and assume that universities maximize a composite of the quality they offer to students, denoted by z, and the economic diversity of their student body, subject to a budget constraint. Quality offered by universities is an abstract concept. The literature has argued that the quality offered by a school is determined both by the quality of the student body and the investments per student done by the school. Dennis Epple and Sieg (2006), for instance, model the objective function of the university as a composite of the average ability of the student body, the investment per student and the inverse of the mean income. They argue that there is empirical and anecdotal evidence that shows that colleges engage in policies to attract low income students. Universities take as given the values of τ, s, R, w and the distribution F. Additionally, we assume that universities set their policies simultaneously and so, the pricing and admission policies set in equilibrium should satisfy the no profitable one shot deviation principle.

University j takes as given $(\tau, r, s, R, w, P^{-j}, \underline{\theta^{-j}})$ and will set the pricing and admission policies $(P^j, \underline{\theta^j})$ in order to solve the following problem:

$$\max_{\{P^j, \underline{\theta^j}, I^j\}} \left(z^j\right)^{\alpha} \left(\sigma_b^j\right)^{1-\alpha} \tag{5}$$

subject to:

$$z^j = \tilde{\theta}^{j^{\alpha_1}} (I^j)^{\alpha_2} \tag{6}$$

$$\tilde{\theta}^j = \int_{\Theta \times B} \theta \cdot e^j(\theta, b) dF(\theta, b) \tag{7}$$

$$I^j \cdot N^j + V^j(N^j) + C^j = P^j \cdot N^j + E^j \tag{8}$$

$$N^j = \int_{\Theta \times B} e^j(\theta, b) dF(\theta, b) \tag{9}$$

where $\tilde{\theta}^j$ is the average ability of the individuals that attend school j. σ_b^j is the inverse of the average income of the student body and reflects the fact that universities care about the diversity in their student body. $e^j(b, \theta)$ indicates with values zero or one if a student with ability θ and wealth b decides to study or not. I^j is the monetary amount that the university invests per student, V^j is a convex cost function, N^j is the size of the student body, C^j is a fixed cost and E^j the university's endowment. Note that the policy P^j does not depend on student's characteristics such as wealth or skills. This is not only a simplifying assumption but also follows closely the case of Colombia where private universities do not price-discriminate students based on ability or wealth. As will be discussed in the relevant section, the extent of financial aid provided by such institutions is very limited in the period of analysis.

3.3 Discussion

Even though the solution to the problem of the university seems simple given that there are only two variables of choice, there are several elements of the model that increase the complexity of the computation. First of all, both policies are interdependent. When a university changes one decision variable -either the price or the admission threshold- this will distort the incentives faced when setting the other policy. For instance, a change in tuition price will not only change the revenue of the university but will change the demand in a way that we expect to see a change in the average ability of the student body. Such a change in the average ability of the student body will affect the marginal productivity of investments made by the university, which in turn will affect its pricing decisions.

Moreover, we need to deal with the fact that in equilibrium no university should have incentives to deviate. Given that both universities make the decision simultaneously and that there are no elements of incomplete information in the model, the relevant equilibrium concept is Nash Equilibrium: no university should have incentives to deviate, given the decisions made by the other university. Note that given the nature of the problem we cannot be sure of the existence of such equilibrium -university payoffs are not continuous- and moreover, uniqueness cannot be guaranteed.

The aforementioned elements make it clear why analyzing the consequences that subsidized loan policies will have in the market of higher education is a complex problem. Let's suppose that the government imposes such policy by subsidizing the interest rate of student loans. The first effect such policy will have is an increase in the number of students going to college. Note, however, that it is also not unreasonable to assume that the quality of the student body will change. This is because people who changed their decision to go to college are either those who were credit constrained or those having low ability levels that now decide to go to college given the decrease in the opportunity cost.

We can expect that after imposing such a policy, households will react by changing their decision of studying and universities should expect a change not only in the size of their student body but also in their quality. Given such changes, universities might want to change the prices charged to their students. This is due to the fact that as the quality of the student body changes, the productivity of investment will also be affected. Additionally, the willingness to pay for educational services is affected by such policy and universities will react to that. Moreover, universities might want to change the admission threshold either to improve the quality of their student body or to attract less able students that are willing to pay more for education. The overall effect depends on how sensitive is the demand for education with respect to the quality of services being provided.

Finally, note that -as said previously- the decisions of universities need to be analyzed in equilibrium. When deciding what is optimal, each college needs to take into account what their competitor is doing in the market and there should be no room for profitable deviations. After imposing a subsidized loan policy we might end up in an equilibrium where one college serves a specific part of the population. For instance one college serves a large demand for students with relatively low levels of ability whereas the other one specializes in providing high quality education for a reduced number of high ability students. Additionally, we can have a symmetric equilibrium where both firms are indistinguishable from one another or one in which only one firm operates in the market.

3.4 Government

We do not model the government as a welfare maximizing agent in the economy. We abstract from this fact and simply analyse the impact of the change in the government policies on the higher education market. However, we do interpret the student loan policy implementation as a way of the government to reduce the existent inefficiency in the educational market.

In a social planner's solution, the efficient outcome would be one in which the high ability individuals decide to study, independent of their wealth. Thus, the role of the student loan policy can be interpreted as a way to reduce the existing inefficiency in the educational sector, although we do not model it as an optimal decision. We assume that the government has a borrowing constraint in the international borrowing markets, so is only able to finance the education of some fraction of the individuals in the economy. For now, we assume that the government finances individuals that have $\theta \geq \theta_{min}$, and of those that can access the loans, subsidizes the interest rate on the loan for those individuals that have $b \leq b_{max}$. The government sets thresholds $\bar{b}$ and θ_0, such that

$$s \cdot (R - 1 - r) \cdot \int_{\Theta_2 \times B_2} \left(e^l(b,\theta) + e^h(b,\theta) \right) \times dF(\theta, b) = \tau \int_{\Theta \times B} b dF(\theta, b) \tag{10}$$

where $\Theta_2 \times B_2 = (\Theta_1 \times B_1) \cap ([\theta_0, 1] \times [0, \bar{b}])$ is the set of households who study and decide to take the subsidy.

Definition 1 (Competitive Equilibrium). *Given a set of government policies, $\tau, s, b_{max}, \theta_{min}$, and prices R, r, w, a competitive equilibrium is a set of university policies $(P^j, \underline{\theta}^j, I^j)_{j=h,l}$ and household's value function $V(\theta, b)$ and policy functions $c(\theta, b), a(\theta, b), e^h(\theta, b), e^l(\theta, b)$, such that:*

1. *Given* $\tau, s, b_{max}, \theta_{min}$, R, r, w, *and university policies* $\{P^j, \underline{\theta^j}, I^j\}_{j=h,l}$, *the value function* $V(\theta, b)$ *solves the household's problem, with* $c(\theta, b)$, $a(\theta, b)$, $e^h(\theta, b)$ *and* $e^l(\theta, b)$ *being the corresponding policy functions.*

2. *For each university* $j = h, l$, *it should hold that given* $\tau, s, b_{max}, \theta_{min}$*; prices,* R, r, w*; policy functions* $c(\theta, b), a(\theta, b), e^h(\theta, b), e^l(\theta, b)$*; and policies from university* $-j$, $(P^{-j}, \theta^{-j}, I^{-j})$*, university* j *chooses policies* $(P^j, \underline{\theta^j}, I^j)$ *that solve the university's problem described in 5-9.*

3. *The government's budget is balanced (equation 10 holds).*

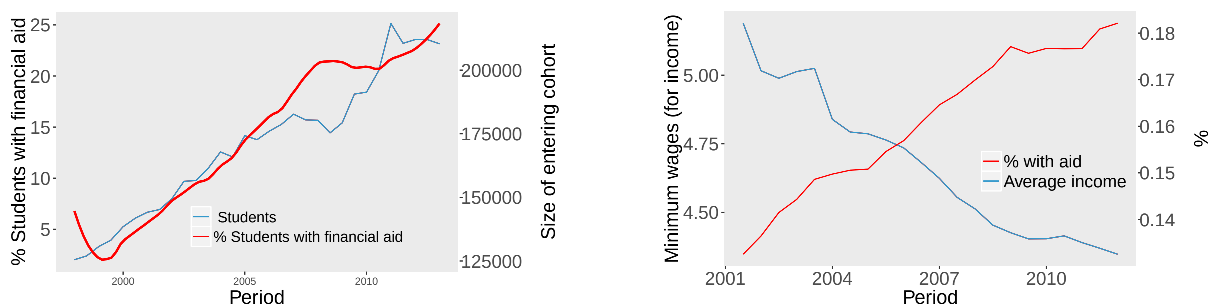
The nature of the problem makes it hard not only to compute the competitive equilibrium but also to show its existence. Note that, by only analyzing the supply side of the market, we cannot be sure that such an equilibrium will exist in this economy. In order to compute the Nash equilibrium of the supply side of the market, we need to find pricing and admission policies that are profit-maximizing given what the policies of the other university.

The computation of such equilibrium is more involved when we note that there is an additional fixed-point problem in the computation of the equilibrium. Universities offer their students a given level of quality that needs to be self-fulfilled: the quality offered by universities will attract certain students to the market but the quality of students going to universities determines the quality offered by universities. It is not possible to use any fixed-point theorem to show existence of a fixed point in this quality self-fulfilling problem given that the necessary assumptions are not satisfied. In particular, note that the fixed-point quality problem is not continuous as whenever the low-quality university offers the same quality as the high one, all students who are beyond the ability threshold will go to the cheapest one, generating a massive exit from one university to the other one, generating a discontinuous jump in the quality being offered.

In order to illustrate this point extensively, we show in appendix A.7 the failure to prove existence of the equilibrium in the case of a linear utility function.

4 The "Revolución Educativa" of Colombia

In the present research, we will use Colombia as a natural experiment of a country that implemented a rapid credit expansion program to alleviate credit constraints. Colombia is a developing country which by the beginning of last decade had low enrollment rates in post-secondary education, and significant differences in enrollment by quintiles of income. As will be argued, the majority of students came from high-income families, and the existence of financial constraints kept high-ability individuals from the lowest quintiles out of the education market. During the last decade, the government engaged into the strategy *Revolucion Educativa*, aimed at increasing the education coverage at all levels. During the decade, there were substantial increases in enrollment and educational credit access (see Figure 6).



(a) Enrollment and % of students with financial aid.

(b) Average income and % of students with financial aid.

Figure 6: Enrollment, income and financial aid.

4.1 Enrollment and inequality

At the beginning of last decade, college enrollment in Colombia was among the lowest in Latin America and a student financial aid system was almost non-existent. In 2000, 23.2% of the people between 18 and 23 years old enrolled in tertiary education, below the enrollment rates of Bolivia, Peru, Brazil, Chile and Venezuela, and very close to the enrollment rates of Mexico. Because of a lack of a well-functioning financial aid system, less than 5%[7] of the entering cohorts had any kind of public or private financial support (World Bank, 2003, 2012). By the end of the decade, the

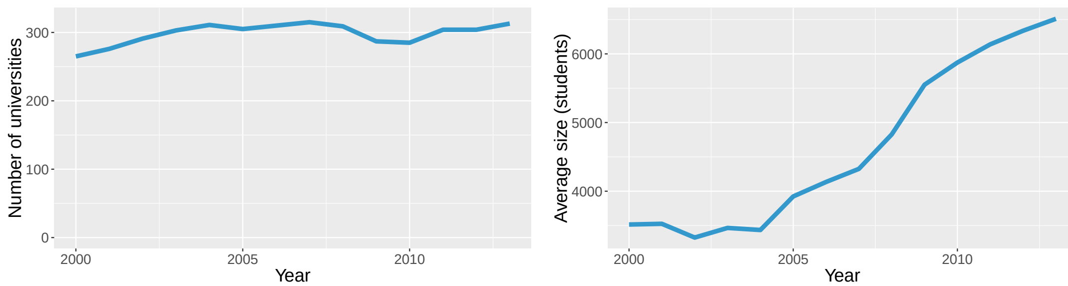
[7] Extracted from the dataset of indicators for tertiary education, SPADIES, from the Ministry of Education.

enrollment rates grew to 37%, and reached 50% in 2015. The fraction of students with some type of credit increased to almost 25% of the entering cohorts (see Figure 6(a)).

Access to education has always been unequal and, despite the fast growth of enrollment, many disparities persist. In 2013, only 45% of the low-income students graduated from high school, and only 25% of them enrolled in tertiary education. Of the high-income households, 60% graduated from high school and 54% of them enrolled in a post-secondary institution (Melguizo et al., 2015). According to World Bank (2003, 2012), the enrollment gap between the lowest and the highest quintiles of wealth widened throughout the decade: in 2001, the enrollement rates were of 8% in the lowest quintile and 41% in the highest, while in 2010 these numbers grew to 10% and 52%, respectively. If quality is taken into account, disparities are even larger as a larger proportion of the low-income students attend non-professional institutions, which have less resources and offer lower expected income in the future. Many theories have been used to explain the low enrollment of low-income students, such as disparities in the quality of public and private high school education, the high costs of tertiary education and the lack of a well-functioning financial aid system (Melguizo et al., 2015).

4.2 Higher Education institutions

The university system in Colombia functions as a monopolistically competitive market in which there are significant institutional barriers to entry, and universities do not have fixed "production capacities", as assumed by Hector Chade and Smith (2014) (Figure 7). There are approximately 300 tertiary education institutions, of which around 190 are universities, and the rest offer non-professional degrees (mainly technical and technological). Despite the growing size of the entering cohorts throughout the decade, the number of institutions remained almost constant, while the average size of each institution doubled, on average. It is important to note that around $45-50\%$ of the total student body is enrolled in private tertiary education institutions (OECD and Bank, 2012). Private institutions do not have any regulations regarding the price or investment per student they offer, although they have to satisfy a minimum quality requirement in terms of the programs and degrees offered. Therefore, the education market in Colombia can be studied as a monopolistically competitive market with barriers to entry and not subject to much government regulation.



(a) Left: number of universities.

(b) Right: average size of universities

Figure 7: Evolution of Higher Education in Colombia

In Colombia, every student that wants to graduate from high school has to present an exam called SABER11 set by the Colombian Institution for Education Evaluation (ICFES), similar to the SAT test in the U.S. Although not every tertiary education institution takes into account the results of the SABER11 in their admission decision, 78% use it as a criterion for admission (OECD and Bank, 2012). As SABER11 has no pass-mark, each institution sets its own minimum threshold for admission. In contrast to what happens in Chile and some European countries, in Colombia there is not any institution that clears the market for admissions, so individuals apply to as many institutions as they like and universities choose their admission standards independently (Melguizo et al., 2015). Although not perfect, the results in the SABER11 exam can be used as a proxy for the quality of the student body at universities. Figure 8 illustrates the average decile of the SABER11 scores of the entering cohorts to tertiary education institutions. Throughout the decade, universities seem to have adjusted their admissions standards in such a way that led to a reduction in the ability of the student body, as measured by relative position in the test scores.

Figure 8: Average decile of ability of entering student body, measured by test scores

4.3 The ACCES Program

To alleviate the low access, in 2002 the government implemented the credit program *Access with Quality to Higher Education*, ACCES, with the support of the World Bank, that massively increased the available credit to students. The credit is awarded to students that have test scores above a threshold set by the government, and covers up to 75% of the tuition for the lowest income students, and up to 50% for the rest. The credit has a subsidized zero-real interest rate for the poorest households, and a real interest rate of 8% for the high-income students. Students that graduate from their programs have twice the time of their study period to repay the loan. The ACCES program has full coverage, in the sense that any student that has test scores on the highest deciles of their region can access this credit line. The test score cut-offs vary by region, to account for disparities in the quality of secondary education across regions with different infrastructure and economic development. Given that the credit is awarded according to regional cutoffs, the disparities in the ability of people accessing the credit are large. The best students from the poorest regions might not have high ability and preparation when compared to the best students of the principal cities, so the credit is not awarded to the highest ability individuals in absolute terms.

Using a regression discontinuity approach, Melguizo et al. (2015) find evidence that the ACCES program had a positive impact on the enrollment rates, especially for individuals that come from poor households. Although the growth in the number of students enrolled in college may have been a consequence of other factors, such as better economic activity, the massive increase in financing seems to be a driving factor of such a trend.

4.4 Product Differentiation in the Market for Higher Education

In this subsection we introduce the dataset constructed to analyze the behavior of colleges before and after the introduction of the subsidized loan program. We use administrative data from the Ministry of Education including the SABER-PRO examination scores of each college. These are major-specific examinations that are mandatory in Colombia in order to receive the equivalent of a Bachelor's degree. Additionally, we use publicly available information scrapped from the internet regarding the academic production of professors, as well as the academic credentials of the professorial body for each university. Moreover, we build information regarding the major-specific tuition charged by each college in order to track its behavior during the last ten years.

The analysis suggests that, after the introduction of the subsidized loan policy, elite institutions engaged in significant efforts to improve the quality of services provided. All the evidence suggest that once the subsidized loan policy was introduced, the gap in quality between elite and non-elite institutions increased. Figure 11 shows how the composition of the entering student body in elite institutions changed during the period of reference when compared to non-elite institutions. When ranked according to the decile in the distribution they are located by the SABER-11 examination score, we find that in 2007, students entering to elite institutions where, on average, located 1.5 deciles above the average student entering to non-elite universities. After five years we see that such gap increased to 1.7 and has remained constant until the last period of data available.

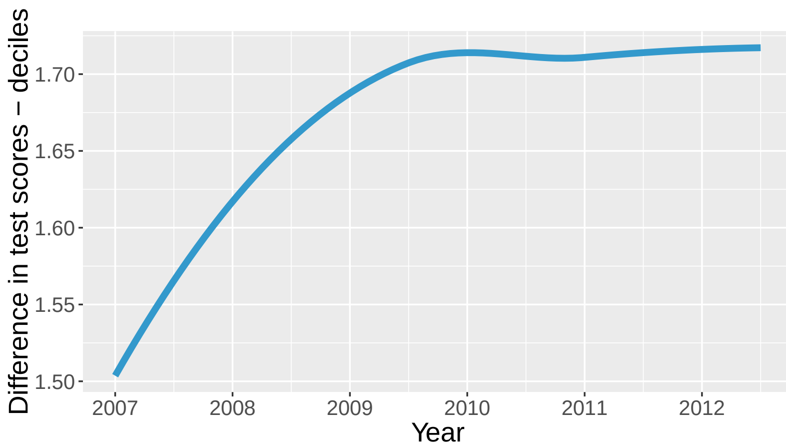
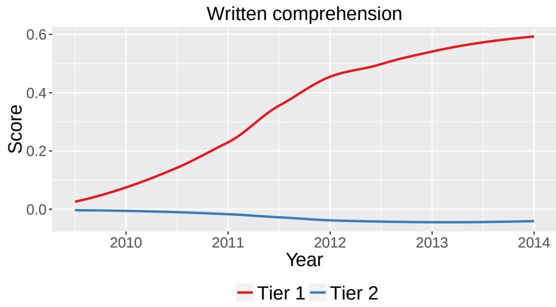


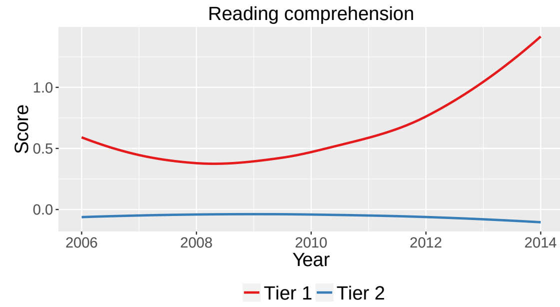
Figure 9: Differences in quality of student body

Note: Differences in the average decile of entering cohort in SABER-11 examination scores. This dataset is constructed using publicly available information provided by the Colombian Ministry of Education on its official website: http://www.mineducacion.gov.co/1759/w3-channel.html

Differences in the quality of student body are also observed when analyzing exit-level examination scores. Figure 10 shows the evolution of average test scores in written comprehension and reading comprehension, for students attending elite and non-elite colleges. The test scores are standardized to be mean zero and standard deviation one for every year in the dataset. Although in 2009 there were negligible differences between test scores of elite and non-elite colleges, we observe that in 2014 the average student graduating from an elite institution would score 60% of a standard deviation above the mean whereas students in non-elite institutions would score slightly below the mean. Taking into account that the average length of a bachelor's degree program lasts 4.5 years, the score for 2014 corresponds to students who were entering in 2008, approximately. This fact is consistent with the scores for reading comprehension presented in panel B of the corresponding figure. Moreover, reading comprehension exams were being done since 2006 and thus we have a longer panel allowing us to infer that no significant changes were observed until cohorts graduating after 2010.



(a) Writing score.



(b) Reading comprehension score.

Figure 10: Quality supplied by colleges.

Note: Test scores are standardized to be mean zero and standard deviation 1 in every year. This information comes from the official statistics provided by the Colombian Ministry of Education.

So far we have provided evidence suggesting that the gap between elite and non-elite institutions, when it comes to the the quality of entering and exiting cohorts, increased after the introduction of the subsidized loan policy. However, the evidence suggest that the behavior of universities changed beyond the quality of their student body. Figure 11 shows that during the same period, elite universities engaged in significant efforts to increase the ratio of professors per students when compared to non-elite institutions. In 2007, the difference in the ratio of professors per student between elite and non-elite institutions, was under 0.02. However, for 2013 the difference more than doubled beyond 0.05.

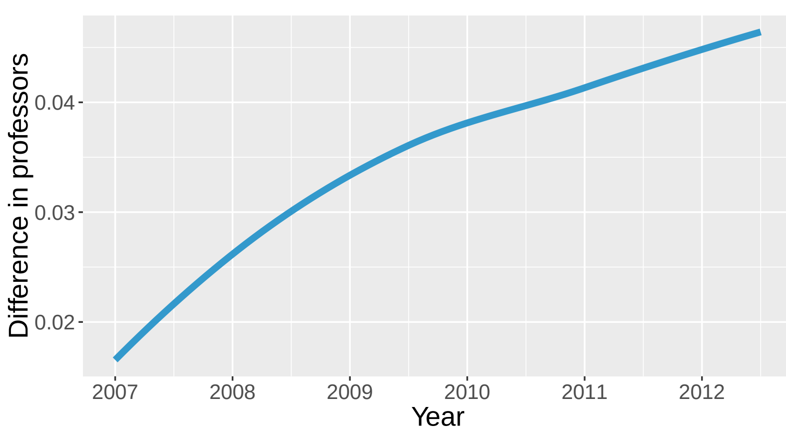


Figure 11: Difference in professors per student

Note: This data is publicly available at the National System for Information on Higher Education website: http://www.mineducacion.gov.co/sistemasdeinformacion/1735/w3-propertyname-2672.html

We can go beyond the gross statistics of professors per student and analyze the academic credentials of the faculty composition of elite and non-elite colleges. In Colombia, it is not uncommon to see new hired faculty whose highest academic credential corresponds to a Bachelor's or a Master's degree. Taking into account this fact, the trend observed in Figure 11 would not imply by itself that elite institutions are making significant efforts to improve the quality of their faculty body. They might be substituting PhD professors by faculty whose highest academic credential is a Bachelor's degree. However, In Figure 12 we find that the professors-student ratio of elite institutions increased when compared to non-elite institutions for every category of professors: those with a PhD, with a Master's degree, and with a Bachelor's degree.

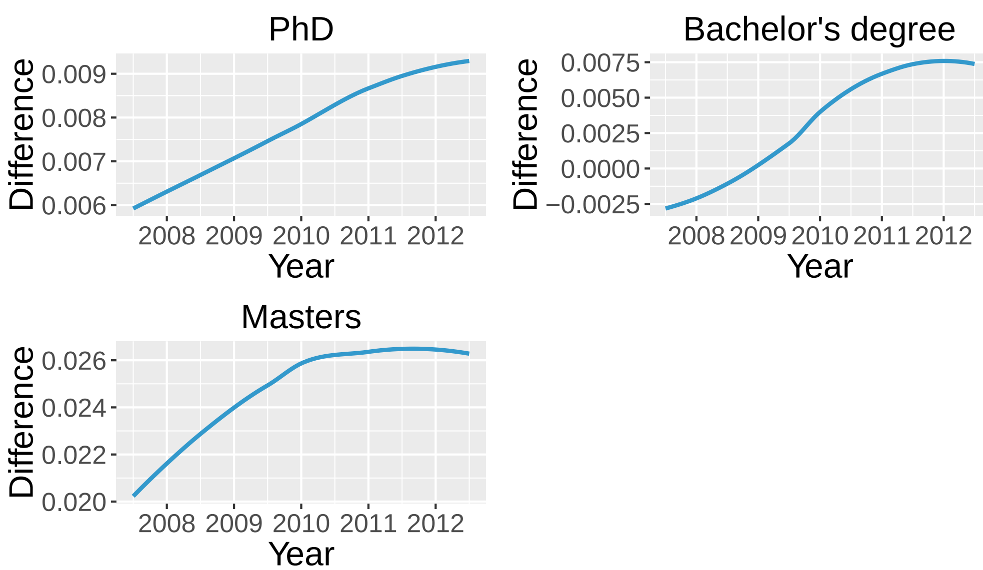
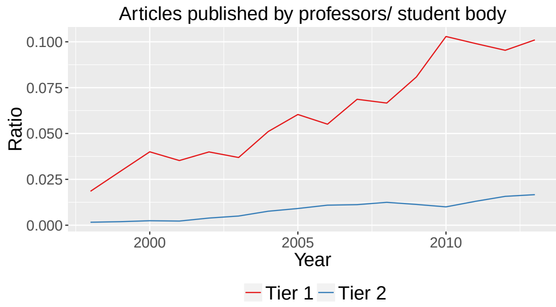
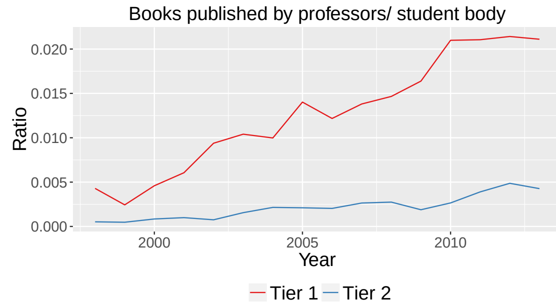


Figure 12: Evolution of Faculty composition. Professors per Student

Finally, the dataset also allows us to analyze the academic production of faculty from every college in Colombia. We construct a dataset of articles published in refereed journals by authors' affiliation as well as total number of books by faculty. The results are presented in Figure 13. When we analyze the academic production per students, as measured by articles and books published, we also find evidence suggesting that the gap in academic production between elite and non-elite universities increased dramatically after the introduction of the subsidized loan policy.



(a) Writing score.



(b) Reading comprehension score.

Figure 13: Gap in academic production.

Note: We constructed this dataset by scrapping information available online at the Administrative Department of Science, Technology and Innovation (Colciencias) http://www.colciencias.gov.co/. A more detailed description of how this dataset was constructed is available in Spanish at http://laramaciudadana.com/universidades.html

Finally, we analyze the evolution of tuition being charged by higher education institutions. Figure 14 illustrates the behavior of the average real price of tuition during the decade, in terms of 2004 pesos. As can be observed, there has been a steady increase in the real price of education throughout the decade for all universities in Colombia. Additionally, the price of the high-quality colleges seems to have peaked at a higher pace for Law, Engineering and Medicine schools. This increasing trend suggests that the Bennett Hypothesis might also be taking place in the Colombian context, given the fast increase in the government provided loans to education.

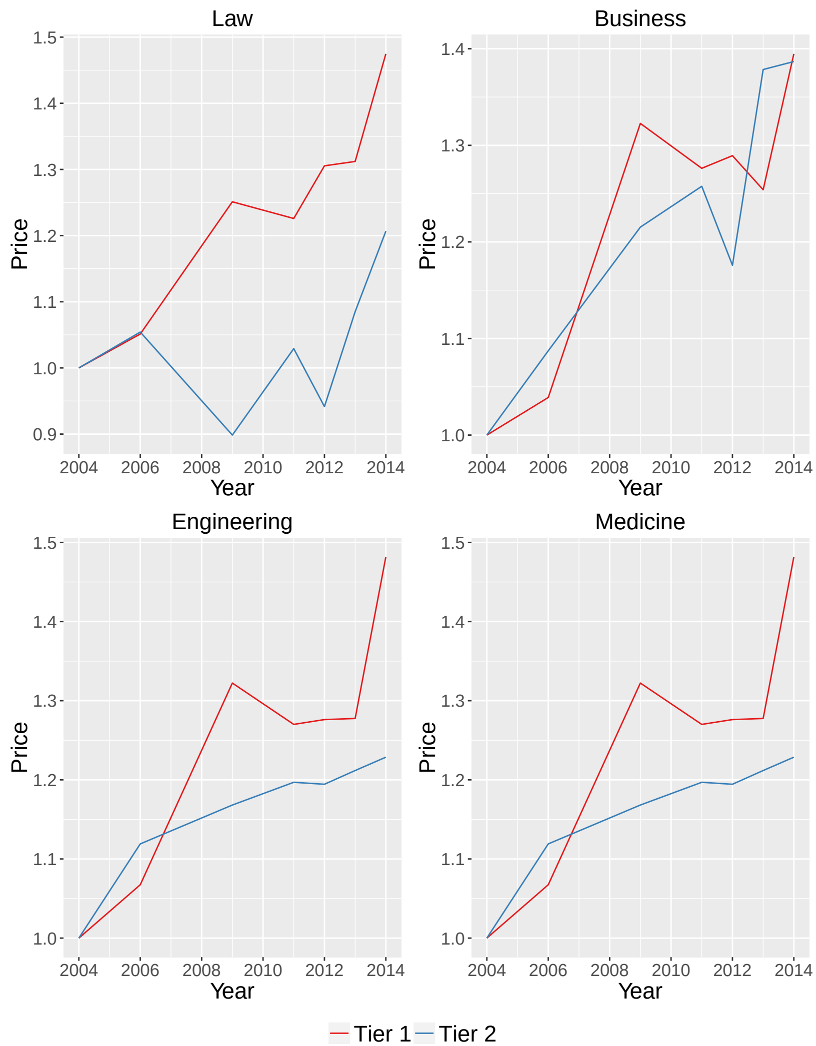


Figure 14: Evolution of average tuition prices over the decade

Finally, we conclude this subsection by summarizing the main findings we observe from the data. We find that after the subsidized loan policy program was implemented in Colombia, the degree of product differentiation between elite and non-elite universities in Colombia increased. We conclude this after analyzing the trend of four key characteristics of universities in Colombia. First, the gap in the quality of student body increased dramatically when analyzing it via entering (SABER-11) or exiting (SABER-PRO) test scores. Second, we observe a gap in the professors per student ratio for every possible category of professorship (PhD, Master's and Bachelor's degree). Third, the gap in academic production, measured as number of peer-reviewed articles and books published, per student, increased during the same period. Finally, we observe that in both, elite and non-elite institutions, there was a significant increase in the tuition being charged for some of the most popular degrees of study. All this evidence is consistent with the fact that after the introduction of subsidized loan policies, the gap in quality between elite and non-elite institutions increased significantly.

5 Numerical Analysis

In this section we perform a numerical exercise to illustrate how the model can rationalize an increase in the quality of elite and non-elite institutions when subsidized loans are introduced. We present how we map the variables of our model to a dataset we construct for higher education in Colombia. We use the case of Colombia since the country experienced a massive policy of subsidized loans for higher in the early 2000's. We construct a dataset of higher education in Colombia and describe how we map the variables into our model.

Our goal is not to prove a causal relationship. Neither are we able to draw quantitative conclusions about the credit expansion in Colombia. Rather than that, we aim to show that the observed variables evolve in a way that is consistent with the mechanisms we suggest through which the introduction of subsidized loans can increase the quality gap between elite and non-elite colleges. We leave the question of causality for future research.

5.1 Evolution of Quality

According to the specifications assumed in the model, we are able to identify the parameters of the wage equation. For this, we will use data on the average wages of graduates from each university in Colombia through 2007-2012, and the minimum wage, as a measure of w, to estimate the parameters of the quality production function of universities. Per-efficiency unit wages are given by:

$$w_h = w \cdot (1 + z_h), \quad w_l = w \cdot (1 + z_l)$$

Where w_h and w_l are the wages of high- and low-quality college graduates, given equilibrium qualities of education z_h and z_l, respectively, and w is the wage of non-skilled labor per-efficiency unit. The quality of education, z, is given by equation (6) in the universities' problem. We have a panel of data for 50 universities in Colombia from 2007 to 2012. We have the average ability of students in the entering cohorts, number of professors per student and average wages during the first year after graduation. For every university i in our sample, the following equation holds:

$$w_i = w \cdot (1 + \kappa \bar{\theta}_i^{\alpha_1} I_i^{\alpha_2})$$

Rearranging and taking logarithms:

$$\log\left(\frac{w_i}{w} - 1\right) = \log \kappa + \alpha_1 \bar{\theta}_i + \alpha_2 I_i$$

Assuming that there is measurement error in the wages of each of the universities, and assuming an exclusion restriction that the measurement error is uncorrelated with the explanatory variables, we can estimate the following equation:

$$\log\left(\frac{w_{i,t}}{w_t} - 1\right) = \log \kappa + \alpha_1 \bar{\theta}_{i,t} + \alpha_2 I_{i,t} + \eta T_{i,t} + \phi_t + \psi_i + \epsilon_{i,t}$$

where T_i is an indicator function that takes the value of one when the university i is a low-quality institution, and zero otherwise. Under this specification, we can estimate possible differences in the technology parameter, κ, between top and second tier schools. In order to isolate possible omitted variable bias, we estimate the above model under three different specifications, with and without time and geographic fixed effects, ϕ_t and ψ_i, respectively.

For the estimation, we constructed a panel of the top 50 universities in Colombia, according to a quality ranking published by the Ministry of Education in 2014[8]. This panel includes data on average wages during the first year after graduation for graduates of every school, as a measure of $w_{i,t}$, the average test scores for the entering cohorts, as a measure of $\theta_{i,t}$, and the number of

[8] The ranking is published in the website of the Ministry of Education, and can be found in the following link: http://www.mineducacion.gov.co/cvn/1665/w3-article-351855.html

Parameter	OLS	OLS	OLS
$\hat{\alpha_1}$	0.211	0.228	0.168
	(0.026)	(0.026)	(0.026)
$\hat{\alpha_2}$	.358	0.478	0.414
	(0.361)	(0.357)	(0.403)
$\hat{\eta}$	-0.029	0.008	-0.046
	(0.047)	(0.043)	(0.046)
$\hat{\log}(\kappa)$	-0.84	-0.957	-0.163
	(0.232)	(0.228)	(0.198)
Time fixed effects		Yes	Yes
City fixed effects			Yes
N	382	382	382
R-squared	0.353	0.444	0.567
Prob > F	0.000	0.000	0.000

Robust standard errors in parenthesis
$***p < 0.01, **p < 0.05, *p < 0.1$

Table 1: Estimates for the quality production function.

professors per student, as a measure for $I_{i,t}$. We also have data on total operational expenditures by each school for 2014. However, with only one year we are not able to construct the evolution of quality of universities over time. Since the number of professors per student are a good indicator of the total expenditures per student, we will use that variable, instead. For the non-skilled labor wages, w_t, we will use the values of the real minimum wage (in 2007 pesos). The average wages of college graduates are strictly above the minimum wage during the period, so the dependent variable is well defined for every college in every period. In addition, we have information about the municipality of the school, to control for regional differences. The results of the estimation are displayed in Table 1.

The estimates show that the elasticities α_1 and α_2 are fairly robust to different specifications and do not change dramatically when including control variables. Moreover, the parameter η is negative in two of the specifications, although non statistically significative. This means that, on average, tier 2 universities have a lower technology parameter, κ, on their quality production function. This will be one of the main differences between tier 1 and tier 2 universities in our numerical exercise.

5.2 Empirical analysis in a two-period model

In order to draw conclusions about the relevance of our model, we map the parameters to values that are relevant to the Colombian context. To achieve this, we map a life-cycle model to a two period model, so the conclusions of Section 3 hold. We follow an approach similar to the one used by Lochner and Monge-Naranjo (2011), but in a discrete time economy. The environment is as follows.

Individuals live for T periods, after which they die with certainty. Individuals start their adult life at $t = 0$, when they must choose whether to attend the low- or high-quality school, or not study at all. Studying lasts for S periods, so those individuals that attend college will not receive any income during $t \in \{0, \ldots, S-1\}$ and have to pay a per-period price of P_j for attending school j. Moreover, during the first S periods individuals are borrowing constrained. Those that decide not to study cannot borrow at all. Those that decide to study, can borrow up to the exogenous limit set by the government student loan policy, $\bar{A}$. After period $S-1$, the individual enters the labor market and earns a per-period wage $\theta w(1+z_j)$, that depends on the quality of the school attended. During periods $S, \ldots, T$ individuals only consume and save. We assume that from period S onwards, individuals enter into perfect financial markets where debt repayments are fully enforced. In this context, individuals can borrow any amount they want.

Clearly, individuals that are not borrowing constrained during their study period will perfectly smooth consumption along the life-cycle. However, those individuals that are constrained during the first S years of life will exhibit a jump in their consumption once they graduate from college. This setting can be easily embedded into the two-period model described in last section, by setting the discount factors and budget constraints appropriately. Namely, the problem for the household becomes:

$$\max_{c,c'} \frac{c^{1-\sigma}}{1-\sigma} + \tilde{\beta}\frac{(c')^{1-\sigma}}{1-\sigma}, \quad s.t.$$

$$c + c'\left(\frac{\Phi_S}{\Phi_0(1+r)^S}\right) + (P_H h + P_l l)\left(\frac{\Phi_r^y}{\Phi_0}\right) =$$

$$w\theta(1-h)(1-l)\left(\frac{\Phi_r^y}{\Phi_0}\right)+w\theta(1+z_j)\left(\frac{\Phi_r^0}{\Phi_0(1+r)^S}\right)j+\frac{b}{\Phi_0}$$

$$a \geq \bar{A}$$

The derivation of the parameters $\tilde{\beta}, \Phi_0, \Phi_S, \Phi_r^o, \Phi_r^y$ is explained in detail in the Appendix A.8. In this environment, all the results from Section 3 hold.

5.3 Parameterization

In our calibration, we set one period to be exactly one year. We will set some parameter values to match the Colombian educational market. All parameter values are reported in Table 2.

We set $S = 5$, so that the individuals that choose to attend a college study during 5 periods, since most professional degrees in Colombia take exactly 5 years. In Colombia, life expectancy at birth is 73.95 years of life[9]. Although the National Statistics Department of Colombia (DANE) does not publish the life expectancy by age, we estimate the life expectancy at 18 years to be 55 more years of life[10]. That is, we set $T = 55$ to match the life expectancy in Colombia for high-school graduates.

We set $\sigma = 2$, which a standard parameter in the literature (Lochner and Monge-Naranjo, 2011). For the real interest rate, we choose $r = 8.9\%$, which is the value for Colombia in 2014 published by the World Bank[11]. We do not claim that this value is representative of developing countries, since the real interest rate for most Latin American countries has a huge variation, ranging from negative values in Argentina (-4.1%) and Venezuela (-14.5% in 2013), to very high values like Brazil (23.5%). We choose $\beta = 0.92$ such that $\beta = 1/(1+r)$. With these parameter values, the discount factor in our two-period model becomes $\tilde{\beta} = 1.89$. This reflects the fact that the post-college period is much longer than the study period, even though individuals discount time at a high rate.

9 See life expectancy tables here.
10 For instance, in the U.S. life expectancy at 18 is only 0.79 more years than life expectancy at birth. Therefore, we will set life expectancy at 18 in Colombia to be 1.05 years above life expectancy at birth, as a conservative estimate.
11 See the real interest rates for all the countries in this link.

As for the university parameters, we use the estimations of Section 5. In particular, we choose $\alpha_1 = 0.211$, $\alpha_2 = 0.358$, $\kappa_l = 0.8$ and $\kappa_h = 0.85$, obtained from the wage regressions displayed in Table 1.

Parameter	Value	Source
Utility and discount		
β	0.97	Literature
σ	2	Literature
r	2%	Colombia
w	2	Normalization
Time parameters		
T	78	Colombia
S	5	Colombia
University parameters		
α_1	0.211	Estimation
α_2	0.358	Estimation
κ_l	1.4	Estimation
κ_h	1.2	Estimation
$E^h - C^h$	-12	Estimation
$E^l - C^l$	-7	Estimation

Table 2: Parameter values

5.4 Results

In this section we show the results of the numerical computation of the equilibria before and after the subsidized-loan policy is implemented. In order to mimic as closely as possible the post-reform equilibrium, we set up a tax rate of 10% used to fund a subsidized loan policy offering credits for higher education for people whose income is below the median income in the economy. The policy implemented in Colombia is designed as a subsidy to the interest rate paid by students. In the model we set up the subsidy in such a way that students that have access to it only have to pay

50% of the interests accumulated in students debts. In addition to having an income below the median, a student who wants to qualify for the policy must have an ability level in the top 30%[12]. Table 3 illustrates the results before and after the implementation of the student loan policies. As can be observed, after the reform there is a widening gap in the quality offered by each university. Elite universities offer a higher quality, while non-elite universities reduce it. There is also a market segmentation, where better students attend the elite institution, and the ability of the students attending the low-quality institutions falls.

		Pre-reform	Post-reform
Elite institutions	Students attending	5,863	9,431
	Average ability of student body	0.48	0.64
	Quality offered	1.01	1.19
Non-elite institutions	Students attending	6,971	6,753
	Average ability of student body	0.41	0.38
	Quality offered	0.53	0.42

Table 3: Equilibria computations

6 Conclusion

Subsidized loan policies have been widely implemented in both developing and developed economies, as a policy tool to increase college attendance. Such policies are particularly relevant in a context where credit constraints explain college attendance decisions and individuals are borrowing constrained. However, when implementing such policies it is important to take into account, not only their effect on the demand side of the market, but also the way they affect the supply side of the market by changing the incentives of the providers of higher education.

We show that subsidized loan policies can distort the incentives of colleges providing services of higher education in a way that can be harmful for a group of households in the economy. Taking into account that the market for higher education operates under a monopolistic competition

[12]The institutional details of the policy implemented in Colombia are fully described in Melguizo et al. (2015)

setting, granting subsidized loans does not generate an expansion in the number of providers of higher education, but rather on the same colleges facing a new set of incentives. As elite institutions unambiguously observe an increase in their demand, they can use their pricing and admission policies to be more selective in their admission process and to spend more per student, which translates into providing better services for their student body. On the contrary, the universities in the low-quality tier will observe a migration to the high-quality group when such policies are implemented. The result is a new equilibrium in the market for higher education where the quality gap between elite and non-elite institutions is widened as a result of the implementation of subsidized loan policies.

Our model is consistent with what we observe in the market for higher education in Colombia: an expansion of the gap in the quality offered by different institutions. In such scenario, subsidized policy loans can make some households worse off as, although the attendance to higher education institutions becomes easier, the gains from attending low-quality universities is not offset by the amount households have to pay in taxes in order to pay for the policy implemented.

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A Appendix A

The problem of the households is:

$$\max_{c,l,h,a} \frac{c^{1-\sigma}}{1-\sigma} + \beta \frac{c'^{1-\sigma}}{1-\sigma}, \quad s.t.$$

$$a + c + hP_h + lP_l = w\theta(1-h)(1-l) + b$$

$$c' = w\theta + w\theta z_h h + w\theta z_l l + (1+r)a$$

A.1 Solution of the unconstrained households:

Proof of Theorem 1. The unconstrained consumptions are:

$$c^N = \frac{\left(\beta(1+r)\right)^{-1/\sigma}\left(w\theta(2+r)+(1+r)b\right)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)},$$

$$c'^N = \frac{\left(w\theta(2+r)+(1+r)b\right)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}$$

$$c^l = \frac{\left(\beta(1+r)\right)^{-1/\sigma}\left(w\theta(1+z_l)+(1+r)b-P_l(1+r)\right)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)},$$

$$c'^l = \frac{\left(w\theta(1+z_l)+(1+r)b-P_l(1+r)\right)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}$$

$$c^h = \frac{\left(\beta(1+r)\right)^{-1/\sigma}\left(w\theta(1+z_h)+(1+r)b-P_h(1+r)\right)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)},$$

$$c'^h = \frac{\left(w\theta(1+z_h)+(1+r)b-P_h(1+r)\right)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}$$

The utilities of each of the options are:

$$u^N = \Phi \times \left(w\theta(2+r)+b(1+r)\right)^{1-\sigma}$$

$$u^l = \Phi \times \left(w\theta(1+z_l) + b(1+r) - P_l(1+r)\right)^{1-\sigma}$$

$$u^h = \Phi \times \left(w\theta(1+z_h) + b(1+r) - P_h(1+r)\right)^{1-\sigma}$$

where

$$\Phi = \left(\frac{1}{1-\sigma}\right)\left(\frac{1}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}\right)^{1-\sigma}\left((\beta(1+r))^{(\sigma-1)/\sigma} + \beta\right)$$

The household's decision of whether and where to study follows a *cut-off* rule on θ, and the decision is independent of initial wealth, b. The cut-offs are:

$$\bar{\theta}_l = \frac{1+r}{w}\left(\frac{P_l}{z_l-(1+r)}\right), \quad \bar{\theta}_h = \frac{1+r}{w}\left(\frac{P_h-P_l}{z_h-z_l}\right)$$

□

A.2 Wealth cutoff rules for households:

Proof of Theorem 2. The debt levels of the unconstrained households are:

$$a^N = \frac{w\theta(1-(\beta(1+r))^{-1/\sigma}) + b}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}$$

$$a^l = \frac{b - P_l - (\beta(1+r))^{-1/\sigma} w\theta(1+z_l)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}$$

$$a^h = \frac{b - P_h - (\beta(1+r))^{-1/\sigma} w\theta(1+z_h)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}$$

Given the exogenous borrowing constraint $\bar{A}$, for a given θ we can construct a cut-off $\bar{b}(\theta)$ on the initial wealth such that individuals with $b < \bar{b}(\theta)$ are constrained and $b \geq \bar{b}(\theta)$ are unconstrained. These are given by:

$$a^N \geq \bar{A} \quad \Leftrightarrow \quad b \geq -\bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1 - (\beta(1+r))^{-1/\sigma})$$

$$a^l \geq \bar{A} \quad \Leftrightarrow \quad b \geq P_l + (\beta(1+r))^{-1/\sigma} w\theta(1+z_l) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

$$a^h \geq \bar{A} \quad \Leftrightarrow \quad b \geq P_h + (\beta(1+r))^{-1/\sigma} w\theta(1+z_h) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

That is, the cut-offs are:

$$\bar{b_u^N}(\theta) = -\bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1 - (\beta(1+r))^{-1/\sigma})$$

$$\bar{b_u^l}(\theta) = P_l + (\beta(1+r))^{-1/\sigma} w\theta(1+z_l) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

$$\bar{b_u^h}(\theta) = P_h + (\beta(1+r))^{-1/\sigma} w\theta(1+z_h) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

□

This subdivides the state space in three subregions, as shown in the following Figure 2.

A.3 Solution of the constrained households:

Next, we have to consider the decision of studying of those households that are constrained. Note that, although if an individual is borrowing constrained when he decides to study, he might prefer to study and not smooth consumption, than not studying and being able to smooth consumption. Therefore, we must compare the utility of studying while being constrained, with the utility of not studying and being unconstrained. The constrained consumptions are given by:

$$c_c^N = w\theta + b + \bar{A}, \quad c_c'^N = w\theta - (1+r)\bar{A}$$

$$c_c^l = b - P_l + \bar{A}, \quad c_c'^l = w\theta(1+z_l) - (1+r)\bar{A}$$

$$c_c^h = b - P_h + \bar{A}, \quad c_c'^h = w\theta(1+z_h) - (1+r)\bar{A}$$

There are three decisions to characterize:

1. Whether to study in l or not study, for individuals that are constrained when studying in l. These individuals will study in l whenever their utility is larger than the unconstrained utility without studying:

$$u(c_c^l) + \beta u(c_c'^l) \geq u(c^N) + \beta u(c'^N)$$

$$\iff \quad u(c_c^l) + \beta u(c_c'^l) - u(c^N) - \beta u(c'^N) \geq 0$$

$$\iff \quad \left(\frac{1}{1-\sigma}\right)(b - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_l) - (1+r)\bar{A})^{1-\sigma} - \Phi \times (w\theta(2+r) + b(1+r))^{1-\sigma} \geq 0$$

2. Whether to study in l or in h, for individuals that are constrained when studying in h but not constrained when studying in l. These individuals will study in h whenever:

$$u(c_c^h) + \beta u(c_c'^h) \geq u(c^l) + \beta u(c'^l)$$

$$\iff \quad u(c_c^h) + \beta u(c_c'^h) - u(c^l) - \beta u(c'^l) \geq 0$$

$$\iff \quad \left(\frac{1}{1-\sigma}\right)(b - P_h + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_h) - (1+r)\bar{A})^{1-\sigma} - \Phi \times \left(w\theta(1+z_l) + b(1+r) - P_l(1+r)\right)^{1-\sigma} \geq 0$$

3. Whether to study in l or in h, for individuals that are constrained when they decide to study in h or l. These individuals will study in h whenever:

$$u(c_c^h) + \beta u(c_c^{'h}) \geq u(c_c^l) + \beta u(c_c^{'l})$$

$$\iff \quad u(c_c^h) + \beta u(c_c^{'h}) - u(c_c^l) - \beta u(c_c^{'l}) \geq 0$$

$$\iff \quad \left(\frac{1}{1-\sigma}\right)(b - P_h + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_h) -$$

$$(1+r)\bar{A})^{1-\sigma} \left(\frac{1}{1-\sigma}\right)(b - P_l + \bar{A})^{1-\sigma} - \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_l) - (1+r)\bar{A})^{1-\sigma} \geq 0$$

The cut-offs that define the college decision for constrained individuals are defined in the following theorem proofs:

Proof of Theorem 3. Define the following function:

$$\begin{aligned} G(\theta, b) &= u(c_c^l) + \beta u(c_c^{'l}) - u(c^N) - \beta u(c^{'N}) \\ &= \left(\frac{1}{1-\sigma}\right)(b - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_l) - (1+r)\bar{A})^{1-\sigma} - \\ &\quad \Phi \times (w\theta^*(2+r) + b(1+r))^{1-\sigma} \end{aligned}$$

Let the function $\bar{b}_c^l(\theta)$ be implictly defined by the equality $G(\theta, \bar{b}_c^l(\theta)) = 0$. By the implicit function theorem:

$$\frac{\partial \bar{b}_c^l(\theta)}{\partial \theta} = -\frac{\partial G/\partial \theta}{\partial G/\partial b}$$

Where:

$$\begin{aligned}
\frac{\partial G}{\partial \theta} &= u'(c_c^l)\frac{\partial c_c^l}{\partial \theta} + \beta u'(c_c^{\prime l})\frac{\partial c_c^{\prime l}}{\partial \theta} - u'(c^N)\frac{\partial c^N}{\partial \theta} - \beta u'(c^{\prime N})\frac{\partial c^{\prime N}}{\partial \theta} \\
&= \beta u'(c_c^{\prime l})w(1+z_l) - u'(c^N)\frac{(\beta(1+r))^{-1/\sigma}w(2+r)}{1+(\beta(1+r))^{-1/\sigma}(1+r)} - \beta u'(c^{\prime N})\frac{w(2+r)}{1+(\beta(1+r))^{-1/\sigma}(1+r)} \\
&= \beta w(1+z_l)u'(c_c^{\prime l}) - \beta w(2+r)u'(c^{\prime N})
\end{aligned}$$

Where the last equality comes from the fact that $u'(c^N) = \beta(1+r)u'(c^{\prime N})$. Similarly:

$$\begin{aligned}
\frac{\partial G}{\partial b} &= \beta u'(c_c^l) - u'(c^N)\frac{(\beta(1+r))^{-1/\sigma}(1+r)}{1+(\beta(1+r))^{-1/\sigma}(1+r)} - \beta u'(c^{\prime N})\frac{(1+r)}{1+(\beta(1+r))^{-1/\sigma}(1+r)} \\
&= u'(c_c^l) - u'(c^N)
\end{aligned}$$

Where, again, the last equality comes from the fact that $u'(c^N) = \beta(1+r)u'(c^{\prime N})$.

$$\implies \quad \frac{\partial \bar{b}_c^l}{\partial \theta} = \frac{\beta w\left((2+r)u'(c^{\prime N}) - (1+z_l)u'(c_c^{\prime l})\right)}{u'(c_c^l) - u'(c^N)} \tag{11}$$

The cut-off θ^* is defined implicitly, such that:

$$(1+z_l)u'\left(w\theta^*(1+z_l) - (1+r)\bar{A}\right) - (2+r)u'\left(\frac{w\theta^*(2+r)+(1+r)b(\theta^*)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}\right) = 0$$

Setting $\partial G/\partial \theta = 0$ gives the result in Theorem 3.

To prove that $\bar{b}_c^l(\theta)$ is a convex function of θ, it suffices to show that $\frac{\partial^2 \bar{b}_c^l}{\partial \theta^2} > 0$:

$$\frac{\partial^2 \bar{b}_c^l}{\partial \theta^2} \geq 0$$

$$\Leftrightarrow \quad \left[(2+r)u''(c'^N)\left(\frac{\partial c'^N}{\partial \theta} + \frac{\partial c'^N}{\partial b}\frac{\partial b}{\partial \theta}\right) - w(1+z_l)^2 u''(c_c'^l)\right] \cdot \left[u'(c_c^l) - u'(c^N)\right]$$

$$- \quad \left[(2+r)u'(c'^N) - (1+z_l)u'(c_c'^l)\right] \cdot \left[u''(c_c^l)\left(\frac{\partial b}{\partial \theta}\right) - u''(c^N)\left(\frac{\partial c^N}{\partial \theta} + \frac{\partial c^N}{\partial b}\frac{\partial b}{\partial \theta}\right)\right] \geq 0$$

Expanding terms, this is true if and only if:

$$\frac{\partial^2 \bar{b}_c^l}{\partial \theta^2} = (2+r)u'(c'^N)u''(c^N)\left(\frac{\partial c^N}{\partial \theta} + \frac{\partial c^N}{\partial b}\frac{\partial b}{\partial \theta}\right) - (2+r)u''(c'^N)\left(\frac{\partial c'^N}{\partial \theta} + \frac{\partial c'^N}{\partial b}\frac{\partial b}{\partial \theta}\right)u'(c^N) - (2+r)u'(c'^N) \tag{12}$$

$$+ \quad (1+z_l)u'(c_c'^l)u''(c_c^l)\left(\frac{\partial b}{\partial \theta}\right) - (2+r)u'(c'^N)u''(c_c^l)\left(\frac{\partial b}{\partial \theta}\right) \tag{13}$$

$$- \quad w(1+z_l)^2 u''(c_c'^l)\left[u'(c_c^l) - u'(c^N)\right] \tag{14}$$

$$+ \quad (2+r)u''(c'^N)\left(\frac{\partial c'^N}{\partial \theta} + \frac{\partial c'^N}{\partial b}\frac{\partial b}{\partial \theta}\right)u'(c_c^l) - (1+z_l)u'(c_c'^l)u''(c^N)\left(\frac{\partial c^N}{\partial \theta} + \frac{\partial c^N}{\partial b}\frac{\partial b}{\partial \theta}\right) \geq 0 \tag{15}$$

In equilibrium, given that households that do not study are unconstrained, the Euler equation holds, so that $u'(c^N) = \beta(1+r)u'(c'^N)$. Taking the derivative of both sides of this equality with respect to θ:

$$u''(c^N)\left(\frac{\partial c^N}{\partial \theta} + \frac{\partial c^N}{\partial b}\frac{\partial b}{\partial \theta}\right) = \beta(1+r)u''(c'^N)\left(\frac{\partial c'^N}{\partial \theta} + \frac{\partial c'^N}{\partial b}\frac{\partial b}{\partial \theta}\right)$$

Using the Euler equation and this equality, the first line above (12) cancels out. Line (13) can be factorized as:

$$(1+z_l)u'(c_c'^l)u''(c_c^l)\left(\frac{\partial b}{\partial \theta}\right) - (2+r)u'(c'^N)u''(c_c^l)\left(\frac{\partial b}{\partial \theta}\right) = u''(c_c^l)\left(\frac{\partial b}{\partial \theta}\right)\left[(1+z_l)u'(c_c'^l) - (2+r)u'(c'^N)\right]$$

Using the expression for $\frac{\partial \bar{b}_c^l}{\partial \theta}$, given by equation (11), this is equal to:

$$\underbrace{-\frac{u''(c_c^l)}{\beta w}}_{>0}\underbrace{\left(\frac{\partial b}{\partial \theta}\right)^2}_{\geq 0}\underbrace{\left[u'(c_c^l)-u'(c^N)\right]}_{>0}\geq 0$$

Line (14) is trivially greater than or equal to zero:

$$\underbrace{-w(1+z_l)^2u''(c_c'^l)}_{>0}\underbrace{\left[u'(c_c^l)-u'(c^N)\right]}_{>0}\geq 0$$

Finally, line (15) can be simplified:

$$\begin{aligned}
&(2+r)u''(c'^N)\left(\frac{\partial c'^N}{\partial \theta}+\frac{\partial c'^N}{\partial b}\frac{\partial b}{\partial \theta}\right)u'(c_c^l)-(1+z_l)u'(c_c'^l)u''(c^N)\left(\frac{\partial c^N}{\partial \theta}+\frac{\partial c^N}{\partial b}\frac{\partial b}{\partial \theta}\right)\\
=\;&(2+r)u''(c'^N)\left(\frac{\partial c'^N}{\partial \theta}+\frac{\partial c'^N}{\partial b}\frac{\partial b}{\partial \theta}\right)u'(c_c^l)-(1+z_l)u'(c_c'^l)\beta(1+r)u''(c'^N)\left(\frac{\partial c'^N}{\partial \theta}+\frac{\partial c'^N}{\partial b}\frac{\partial b}{\partial \theta}\right)\\
=\;&u''(c'^N)\left(\frac{\partial c'^N}{\partial \theta}+\frac{\partial c'^N}{\partial b}\frac{\partial b}{\partial \theta}\right)\left[(2+r)u'(c_c^l)-\beta(1+r)(1+z_l)u'(c_c'^l)\right]
\end{aligned}$$

The second term can be written as:

$$\begin{aligned}
\left(\frac{\partial c'^N}{\partial \theta} + \frac{\partial c'^N}{\partial b}\frac{\partial b}{\partial \theta}\right) &= \left(\frac{w(2+r)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)} + \frac{(1+r)}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}\left(\frac{\partial b}{\partial \theta}\right)\right) \\
& \left(\frac{1}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}\right)\cdot\left[w(2+r)+(1+r)\left(\frac{\partial b}{\partial \theta}\right)\right] \\
& \underbrace{\left(\frac{1}{1+\left(\beta(1+r)\right)^{-1/\sigma}(1+r)}\right)}_{:=\Psi}\cdot\left[w(2+r)+(1+r)\left(\frac{\beta w\left((2+r)u'(c'^N)-(1+z_l)u'(c_c'^l)\right)}{u'(c_c^l)-u'(c^N)}\right)\right] \\
& \Psi\cdot\left[\frac{w(2+r)u'(c_c^l)-w(2+r)u'(c^N)+w(2+r)\beta(1+r)u'(c'^N)-\beta w(1+r)(1+z_l)u'(c_c'^l)}{u'(c_c^l)-u'(c^N)}\right] \\
& \Psi\cdot\left[\frac{w(2+r)u'(c_c^l)-\beta w(1+r)(1+z_l)u'(c_c'^l)}{u'(c_c^l)-u'(c^N)}\right]
\end{aligned}$$

So line (15) is equal to:

$$\begin{aligned}
& u''(c'^N)\left(\frac{\partial c'^N}{\partial \theta} + \frac{\partial c'^N}{\partial b}\frac{\partial b}{\partial \theta}\right)\left[(2+r)u'(c_c^l)-\beta(1+r)(1+z_l)u'(c_c'^l)\right] \\
= \; & u''(c'^N)\cdot\Psi\cdot w\left[\frac{(2+r)u'(c_c^l)-\beta(1+r)(1+z_l)u'(c_c'^l)}{u'(c_c^l)-u'(c^N)}\right]\left[(2+r)u'(c_c^l)-\beta(1+r)(1+z_l)u'(c_c'^l)\right] \\
= \; & u''(c'^N)\cdot\Psi\cdot w\left[\frac{1}{u'(c_c^l)-u'(c^N)}\right]\left[(2+r)u'(c_c^l)-\beta(1+r)(1+z_l)u'(c_c'^l)\right]^2 \geq 0
\end{aligned}$$

□

A.4 Proof of Theorem 4

Proof of Theorem 4. The proof is similar to Proof A.3. Define:

$$G^*(\theta, b) = \left(\frac{1}{1-\sigma}\right)(b - P_h + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_h) - (1+r)\bar{A})^{1-\sigma} -$$

$$\Phi(w\theta^*(1+z_l) + b(1+r) - P_l(1+r))^{1-\sigma} \tag{16}$$

and setting $\partial G/\partial\theta = 0$ gives the result in Theorem 4.

□

A.5 Proof of Theorem 5

Proof of Theorem 5. The boundaries $\bar{b}_c^h(\theta)$ and $\bar{b}_c^l(\theta)$ are implicitly defined by:

$$G_l(\theta, b) = u(c_c^l) + \beta u(c_c^{'l}) - u(c^N) - \beta u(c^{'N}) = 0$$

$$G_l(\theta, b) = u(c_c^h) + \beta u(c_c^{'h}) - u(c^l) - \beta u(c^{'l}) = 0$$

By implicit function theorem, $\partial\bar{b}_c^j(\theta)/\partial\bar{A} = -\frac{\partial G_j/\partial\bar{A}}{\partial G_j/\partial b}$.

$$\frac{\partial G_j}{\partial\bar{A}} = u'(c_c^j)\frac{\partial c_c^j}{\partial\bar{A}} + \beta u'(c_c^{'j})\frac{\partial c_c^{'j}}{\partial\bar{A}}$$

$$= u'(c_c^j) - \beta(1+r)u'(c_c^{'j}) > 0$$

Where the last inequality is satisfied because the individuals are borrowing constrained when attending university j, so the Euler equation is not satisfied. As shown in the proof of Theorem 3:

$$\frac{\partial G_l}{\partial b} = u'(c_c^l) - u'(c^N) > 0$$

$$\frac{\partial G_h}{\partial b} = u'(c_c^h) - u'(c^l) > 0$$

The first inequality follows because $\bar{b}_c^l$ is defined such that $u(c_c^l) + \beta u(c_c^{\prime l}) = u(c^N) + \beta u(c^{\prime N})$. In equilibrium, $u'(c^N) = \beta(1+r)u'(c^{\prime N})$ and $u'(c_c^l) > \beta(1+r)u'(c_c^{\prime l})$. Assume $u'(c^N) = \beta(1+r)u'(c^{\prime N}) \geq u'(c_c^l) > \beta(1+r)u'(c_c^{\prime l})$. Given that u is strictly concave, u' is strictly decreasing, so this implies that $c_c^l \geq c^N$ and $c_c^{\prime l} > c^{\prime N}$. Give that u is strictly increasing, this contradicts the fact that $u(c_c^l) + \beta u(c_c^{\prime l}) = u(c^N) + \beta u(c^{\prime N})$. The proof of the second inequality is analogous. This means that:

$$\frac{\partial \bar{b}_c^j(\theta)}{\partial \bar{A}} = -\frac{\partial G_j / \partial \bar{A}}{\partial G_j / \partial b} < 0$$

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If $P_h > P_l$ and $z_h > z_l$, then $c_c^l > c_c^h$, $c_c^{\prime h} > c_c^{\prime l}$, and $c^N < c^l$, so:

$$-\frac{\partial G_h}{\partial \bar{A}} = -u'(c_c^h) + \beta(1+r)u'(c_c^{\prime h}) < -u'(c_c^l) + \beta(1+r)u'(c_c^{\prime l}) = -\frac{\partial G_l}{\partial \bar{A}}$$

$$\frac{\partial G_h}{\partial b} = u'(c_c^h) - u'(c^l) > u'(c_c^l) - u'(c^N) = \frac{\partial G_l}{\partial b}$$

So:

$$\frac{\partial \bar{b}_c^h(\theta)}{\partial \bar{A}} < \frac{\partial \bar{b}_c^l(\theta)}{\partial \bar{A}} < 0$$

This completes the proof of the first part of the theorem.

Since $\partial G/\partial b > 0$, the first result follows.

For the second result, note that:

$$\begin{aligned}
\frac{\partial b}{\partial \bar{A}\partial \theta} = \frac{1}{(\cdot)^2} & \Big[\Big(\sigma\beta(1+r)w(1+z_l)(w\theta(1+z_l) - (1+r)\bar{A})^{-(1+\sigma)} \Big) \\
& \cdot \big((1+r)((b - P_l + \bar{A})^{-\sigma} - \Phi(1-\sigma)(1+r)(w\theta(2+r) + b(1+r))^{-\sigma}) \big) \Big] \\
+ \frac{1}{(\cdot)^2} & \big[\big((b - P_l + \bar{A})^{-\sigma} + \beta(1+r)(w\theta(1+z_l) - \bar{A}(1+r))^{-\sigma} \big) \\
& \cdot \big(\sigma\Phi(1-\sigma)w(1+r)(2+r)(w\theta(2+r) + b(1+r))^{-\sigma} \big) \big] \\
\geq 0 &
\end{aligned} \tag{17}$$

This proves Theorem 5.

□

Prueba de

A.6 Computation of Nash Equillibrium

In this section we will describe the algorithm used to compute the Nash Equilibrium between elite and non-elite universities. The Nash Equilibrium is composed by a tuple $(P_h^*, \underline{\theta_h}^*, P_l^*, \underline{\theta_l}^*)$ such that:

$$(P_i^*, \underline{\theta_i}^*) \in \arg \max_{(P_i, \underline{\theta_i}) \in \mathcal{R}^+ \times [0,1]} \left(z_i(P_i, \underline{\theta_i}, P_{-i}^*, \underline{\theta_{-i}}^*) \right)^{\alpha} \left(\sigma_{b,i}(P_i, \underline{\theta_i}, P_{-i}^*, \underline{\theta_{-i}}^*) \right)^{1-\alpha} \tag{18}$$

Note that the problem defined in 18 involves solving for a fixed point nested within another fixed point problem. In particular, the universities will offer a given level of z_l, z_h to the households and, conditional on such offer households will demand education services that need to fulfill the promised levels of z_l, z_h. This implies that when solving for the optimal of the universities we need to take into account that the offered level of productivities need to be satisfied by the demand of educational services. The full procedure to find the Equilibrium is described below:

Computation of the Nash Equilibrium

1. Start algorithm with some initial guess $\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g} \rangle$. Set $E = 10$.

2. Find $\langle P_h^T, \underline{\theta_h^T} \rangle \in \arg\max_{(P_h, \underline{\theta_h}) \in \mathcal{R}^+ \times [0,1]} \left(z_h(P_h, \underline{\theta_h}, P_l^g, \underline{\theta_l}^g) \right)^{\alpha} \left(\sigma_{b,h}(P_h, \underline{\theta_h}, P_l^g, \underline{\theta_l}^g) \right)^{1-\alpha}$

   (a) Set $\langle P_h^r, \underline{\theta_h^r} \rangle = \langle P_h^g, \underline{\theta_h^g} \rangle$

   (b) Given $\langle P_h^r, \underline{\theta_h^r}, P_l^g, \underline{\theta_l^g} \rangle$, go to 5. to compute $\langle z_h, z_l \rangle$

   (c) Given $S1 = \langle P_h^r, \underline{\theta_h^r}, P_l^g, \underline{\theta_l^g}, z_h, z_l \rangle$ compute the objective function of the university $H(S1)$.

   (d) Update for a new guess of the optimal $\langle P_h^r, \underline{\theta_h^r} \rangle = \langle P_h^{new}, \underline{\theta_h^{new}} \rangle$ according to some rule.

   (e) Repeat $(b) - (d)$ until optimal $\langle P_h^T, \underline{\theta_h^T} \rangle$ is found

3. Find $\langle P_l^T, \underline{\theta_l^T} \rangle \in \arg\max_{(P_l, \underline{\theta_l}) \in \mathcal{R}^+ \times [0,1]} \left(z_l(P_h^g, \underline{\theta_h^g}, P_l, \underline{\theta_l}) \right)^{\alpha} \left(\sigma_{b,l}(P_h^g, \underline{\theta_h^g}, P_l, \underline{\theta_l}) \right)^{1-\alpha}$

   (a) Set $\langle P_l^r, \underline{\theta_h^l} \rangle = \langle P_l^g, \underline{\theta_l^g} \rangle$

   (b) Given $\langle P_h^g, \underline{\theta_h^g}, P_l^r, \underline{\theta_l^r} \rangle$, go to 5. to compute $\langle z_h, z_l \rangle$

   (c) Given $S1 = \langle P_h^g, \underline{\theta_h^g}, P_l^r, \underline{\theta_l^r}, z_h, z_l \rangle$ compute the objective function of the university $L(S1)$.

   (d) Update for a new guess of the optimal $\langle P_l^r, \underline{\theta_l^r} \rangle = \langle P_l^{new}, \underline{\theta_l^{new}} \rangle$

   (e) Repeat $(b) - (d)$ until optimal $\langle P_l^T, \underline{\theta_l^T} \rangle$ is found

4. Set $E = ||\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g} \rangle - \langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T} \rangle||$. If E is smaller than a tolerance level, stop the algorithm, the NE is given by the tuple $\langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T} \rangle$. Otherwise, set $\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g} \rangle = \langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T} \rangle$ and go to 2.

5. Computation of $\langle z_h, z_l \rangle$ given $\langle P_h, \underline{\theta_h}, P_l, \underline{\theta_l} \rangle$

   (a) Start algorithm with some initial guess $\langle z_h^g, z_l^g \rangle$ and set $\varepsilon = 10$

   (b) Given $\langle P_h, \underline{\theta_h}, P_l, \underline{\theta_l} \rangle$, the guess $\langle z_h^g, z_l^g \rangle$ and ...the policy functions of the households, compute the realized values of $\langle z_h^r, z_l^r \rangle$

   (c) set $\varepsilon = (z_h^r - z_h^g)^2 + (z_l^r - z_l^g)^2$.

   (d) If ε is smaller to a tolerance level, the algorithm is complete. Otherwise, set $\langle z_h^g, z_l^g \rangle = \langle z_h^r, z_l^r \rangle$ and go to (b).

A.7 Analysis in the linear case

In order to get a clear idea of how credit constraints affect the market for higher education, we illustrate the linear case where $\sigma = 0$. Furthermore, we need to distinguish scenarios where households would like to substitute future for current consumption and the other way around. This is given by the inequality $\beta(1 + r) < 1$. Whenever this inequality is satisfied, households would prefer to get as much debt during the first period. The opposite case, when $\beta(1+r) \geq 1$ will motivate households to save as much as possible given that the returns to savings, in terms of utility, are more than one to one.

Case 1.$\beta(1+r) \geq 1$

In this case, households will prefer to save as much as they want and then the value functions for each case (not study, study in low quality university or study in high quality university) are given by:

$$V^N(b,\theta) = \beta\left[b(1-\tau)(1+r) + w\theta(2+r)\right] \tag{19}$$

$$V^j(b,\theta) = \beta(1+r)(b(1-\tau) - P^j) + \beta w\theta(1+z^j) \tag{20}$$

The value function for households going to the low quality university is only defined whenever they can afford it. That is, whenever $P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w\theta(1+z^l)}{1+r}\}$. In particular, consider the case where $P_l - b(1-\tau) \leq 0$. If this holds, then households are able to afford the price of education with their income after taxes and thus we have no concerns about they not getting enough debt to fund their education.

However, when students should get positive debt in order to attend the low quality university, the amount of debt should satisfy two constraints:

$$P_l - b(1-\tau) \leq \bar{A} \tag{21}$$

$$P_l - b(1-\tau) \leq \frac{w\theta(1+z^l)}{1+r} \tag{22}$$

The constraint given in 21 states that the amount of debt students get should not exceed the upper limit given exogenously in the economy. The inequality given in 22 guarantees that students have enough funds to get the necessary debt to attend college. The two aforementioned inequalities give bounds in b and θ for students to being able to pay the tuition in the low quality college:

$$b \geq b_{p_l} = \frac{P_l - \bar{A}}{1-\tau} \tag{23}$$

$$b \geq L(\theta) = \frac{P_l}{1-\tau} - \frac{w\theta(1+z^l)}{(1-\tau)(1+r)} \tag{24}$$

Now, for households with state variables (b, θ) such that low quality education is affordable, we can define the value of going to the low university as:

$$V^L(b,\theta) = \beta\left[(b(1-\tau) - Pl)(1+r) + w\theta(1+z^l)\right] \tag{25}$$

Similarly, in order to be able to go to the high quality institutions, it should be the case that:

$$b \geq b_{p_h} = \frac{P_h - \bar{A}}{1-\tau} \tag{26}$$

$$b \geq H(\theta) = \frac{P_h}{1-\tau} - \frac{w\theta(1+z^h)}{(1-\tau)(1+r)} \tag{27}$$

For those households, we can define the value of going to the high quality college as:

$$V^H(b,\theta) = \beta\left[(b(1-\tau) - P_h)(1+r) + w\theta(1+z^h)\right] \tag{28}$$

Consider the case of a person who is deciding whether to go to the low quality college or not study. In such case, granted that he could afford to pay tuition, he will decide to attend whenever $V^L(b,\theta) \geq V^N(b,\theta)$. This implies that the decision will be to go to the low quality college whenever:

$$\theta_l \geq \theta_L = \frac{P_l(1+r)}{w[z^l - r - 1]} \tag{29}$$

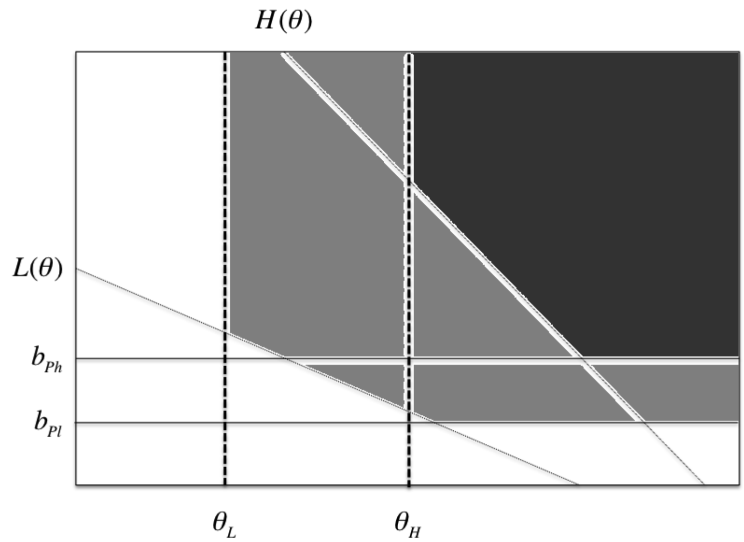

Figure 15: Representation of the education decisions on the state space.

Similarly, when a person is deciding whether to go to the high quality college or to the low quality one, and granted he could afford both, the relevant decision rule will be to go to the high quality college whenever $V^H(b,\theta) \geq V^L(b,\theta)$. This inequality generates the decision rule of going to college whenever:

$$\theta \geq \theta_H = \frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \tag{30}$$

The decision rules can be represented in the state space according to the following graph:

Note that we can express N^H in terms of elements that we have found previously:

$$N^H = \int_{\theta^H}^{\theta^{Ih}} \int_{H(\theta)}^{\bar{b}} dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Ph}}^{\bar{b}} dF(b,\theta) \tag{31}$$

where $\bar{b}$ is the maximum level of bequests in the state space and

$$\theta^{Ih} = \frac{(1+r)\bar{A}}{(1+z^h)w} \tag{32}$$

For the sake of simplicity, we will assume a uniform distribution for (b,θ). As long as $P^h > P^l$ and $z^h > z^l$ we can express the measure of people going to the high quality university as:

$$N^H = \frac{1}{\bar{b}}\left[\left(b - \frac{P_h}{1-\tau}\right)\left(\frac{(1+r)\bar{A}}{(1+z^h)w} - \frac{(P_h - P_l)(1+r)}{w(z^h - z^l)}\right)\right. \tag{33}$$

$$\left. + \frac{w(1+z^h)}{(1-\tau)(1+r)}\left[\left(\frac{(1+r)\bar{A}}{w(1+z^h)}\right)^2 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)}\right)^2\right]\right]$$

Similarly, the average level of skills of people attending such college is given by:

$$\tilde{\theta}^H = \frac{1}{\bar{b}}\left[\left(\left(\frac{(1+r)\bar{A}}{(1+z^h)w}\right)^2 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)}\right)^2\right)\left(\frac{\bar{b}}{2} - \frac{P_h}{2(1-\tau)}\right) + \right.$$

$$\frac{w(1+z^h)}{3(1-\tau)(1+r)}\left[\left(\frac{(1+r)\bar{A}}{w(1+z^h)}\right)^3 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)}\right)^3\right] +$$

$$\left.\frac{1}{2}\left[\bar{b} - \frac{\bar{A}}{1-\tau} + \frac{P_h}{1-\tau}\left(1 - \left(\frac{(1+r)\bar{A}}{1(1+z^h)}\right)^2\right)\right]\right] \tag{34}$$

We can express the relevant variables for low quality college, granted $P_h > P_l$ and $z_h > z_l$, as:

$$N^L = \int_{\theta_L}^{\theta_H}\int_{L(\theta)}^{1} dF(b,\theta) + \int_{\theta_H}^{\theta^{Il}}\int_{L(\theta)}^{H(\theta)} dF(b,\theta) +$$

$$\int_{\theta^{Il}}^{\theta^{Ih}}\int_{b_{Pl}}^{H(\theta)} dF(b,\theta) + \int_{\theta^{Ih}}^{1}\int_{b_{Pl}}^{b_{Ph}} dF(b,\theta) \tag{35}$$

$$\tilde{\theta}^L = \int_{\theta_L}^{\theta_H}\int_{L(\theta)}^{1} \theta dF(b,\theta) + \int_{\theta_H}^{\theta^{Il}}\int_{L(\theta)}^{H(\theta)} \theta dF(b,\theta) +$$

$$\int_{\theta^{Il}}^{\theta^{Ih}}\int_{b_{Pl}}^{H(\theta)} \theta dF(b,\theta) + \int_{\theta^{Ih}}^{1}\int_{b_{Pl}}^{b_{Ph}} \theta dF(b,\theta) \tag{36}$$

$$\mu_{b^L} = \int_{\theta_L}^{\theta_H} \int_{L(\theta)}^{1} bdF(b,\theta) + \int_{\theta_H}^{\theta^{Il}} \int_{L(\theta)}^{H(\theta)} bdF(b,\theta) +$$

$$\int_{\theta^{Il}}^{\theta^{Ih}} \int_{b_{Pl}}^{H(\theta)} bdF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Pl}}^{b_{Ph}} bdF(b,\theta) \tag{37}$$

It is important to note that throughout this analysis we have not implemented the fact that both colleges are able to set a threshold rule such that people with a level of skills below such threshold will not be admitted. In such a case, we will simply modify the regions of integration to consider that only people with ability beyond the threshold will be able to attend.

Existence of equilibrium

The expressions found in 33, 34, 35 and 36 can be used to express the necessary conditions that the offered qualities need to satisfy in equilibrium. In particular, we need to find z^h, z^l such that:

$$\begin{bmatrix} z^h \\ z^l \end{bmatrix} = \begin{bmatrix} \kappa^h \left(\tilde{\theta}^h(\underline{\theta^h}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_1} \left(I(\underline{\theta^h}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_2} \\ \kappa^l \left(\tilde{\theta}^h(\underline{\theta^l}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_1} \left(I(\underline{\theta^h}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_2} \end{bmatrix} \tag{38}$$

We need to prove existence of a fixed point in the qualities offered by universities before proving the existence of the Nash Equilibrium. Note, however, that difficulty arises in this point given the fact that there is no natural way to bound the set of qualities offered by the universities. Additionally, note that equations 33, 34, 35 are not continuous in $z^h = z^l$. The inability of proving the existence of a fixed point in the qualities offered by universities shows that it is not possible to prove existence of the Nash Equilibrium. We rely purely on the computational analysis to find a Nash Equilibrium in this case that might not be unique.

Case 2.$\beta(1+r) < 1$

This case is more involved as households value more current consumption than future and will try to get as much debt as possible. The difficulty arises as even when students can afford to pay college, they might be constrained given that they want to substitute future by current consumption. Additionally, we need to establish which is the relevant constraint that households face when getting the desired level of debt, either the exogenously given level of credit constraint or they reach a point where they can't fund the debt with their resources.

We start analyzing the case of a person who is not going to university. In this case, the person will get as much debt as possible and he will be constrained whenever $\frac{w\theta}{1+r} > \bar{A}$. If this is the case, the person will get the maximum level of debt $\bar{A}$. Taking into account this case when computing the value of not going to college, we see that:

$$V^N(b,\theta) = \begin{cases} b(1-\tau) + w\theta\frac{2+r}{1+r} \text{ if } \theta \leq \frac{\bar{A}(1+r)}{w} \\ b(1-\tau) + w(\theta)(1+\beta) + \bar{A}[1-\beta(1+r)] \text{ if } \theta > \bar{A}\frac{1+r}{w} \end{cases} \tag{39}$$

Now, let's consider a household that goes to the low-quality university. Evidently, the value function will only be defined for the case when it is possible to pay tuition price via endowment or debt. For people whose income is below the tuition price ($b(1-\tau) < P_l$) and who are constrained either by the exogenous level $\bar{A}$ or by their earning capacity $\frac{w\theta(1+z^l)}{1+r}$, the value of going to the low quality college will not be defined.

An individual who is not constrained and takes as much debt as he can, will derive utility given by $b(1-\tau) - P_l + \frac{w\theta(1+z^l)}{1+r}$. The first term, $b(1-\tau) - P_l$ corresponds to net income after tuition and the remaining part $\frac{w\theta(1+z^l)}{1+r}$ is simply the amount they will make in the second period taken to the present value of the first period.

If the net income after tuition is negative, an individual will not be credit constrained so long as:

$$P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w\theta(1+z^l)}{1+r}\} \tag{40}$$

However, it is possible to have individuals who are borrowing constrained even if the net income after tuition is positive. These individuals are those who would like to borrow against their future income, given that current consumption is more valuable than future consumption, but they are not able to borrow as much as they want given the exogenous limit $\bar{A}$. Those are individuals such that:

$$\frac{w\theta(1+z^l)}{(1+r)} < \bar{A} \tag{41}$$

and they are forced to borrow no more than $\bar{A}$. This implies that we can define the value of going to low-quality college as:

$$V^L(b,\theta) = \begin{cases} b(1-\tau) - P_l + \frac{w\theta(1+z^l)}{1+r} \text{ if } \begin{cases} b(1-\tau) - P_l \geq 0 \quad \theta \leq \frac{\bar{A}(1+r)}{w(1+z^l)} \\ \text{or} \\ b(1-\tau) - P_l < 0 \quad P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w(\theta)(1+z^l)}{1+r}\} \end{cases} \\ b(1-\tau) - P_l + \bar{A}[1-\beta(1+r)] + w\beta(1+z^l) \text{ if } b(1-\tau) - P_l > 0 \text{ and } \theta > \frac{\bar{A}(1+r)}{w(1+z^l)} \end{cases} \tag{42}$$

Finally, doing the same analysis but with P_h and z^h we can find the value of going to the high quality college:

$$V^H(b,\theta) = \begin{cases} b(1-\tau) - P_h + \frac{w\theta(1+z^h)}{1+r} \text{ if } \begin{cases} b(1-\tau) - P_h \geq 0 \quad \theta \leq \frac{\bar{A}(1+r)}{w(1+z^h)} \\ \text{or} \\ b(1-\tau) - P_h < 0 \quad P_h - b(1-\tau) \leq \min\{\bar{A}, \frac{w(\theta)(1+z^h)}{1+r}\} \end{cases} \\ b(1-\tau) - P_h + \bar{A}[1-\beta(1+r)] + w\beta(1+z^h) \text{ if } b(1-\tau) - P_h > 0 \text{ and } \theta > \frac{\bar{A}(1+r)}{w(1+z^h)} \end{cases} \tag{43}$$

A.8 Life-cycle Model

In this section we embed a life-cycle model into a two-period model, so our calibration of Section 5.2 is realistic. We solve the household's problem in two parts: 1) during the study periods, $t = 0, \ldots, S-1$, and 2) after college age, $S, \ldots, T$, and leave the problem expressed as a two-period maximization problem in which households decide how much to save for post-college periods.

First, we start by solving the post-college optimization problem. We assume that after college graduation, individuals enter perfect financial markets, so there is perfect consumption smoothing. The problem of the households is:

$$\max_{c_t} \sum_{t=S}^{T} \beta^{t-S} \frac{c_t^{1-\sigma}}{1-\sigma}, \quad s.t.$$

$$c_S = b + a_{S+1} + w(1+z_j)\theta$$

$$c_t + a_t(1+r) = a_{t+1} + w(1+z_j)\theta, \quad t \in \{S, \dots, T\}$$

where a_{t+1} is the debt at period t to be repaid next period, and b are the savings that the individual carries from the college years. In here, we assume that there are no borrowing constraints, since households enter perfect financial markets. Solving this problem, yields the present value budget constraint in period S:

$$\sum_{t=S}^{T} \frac{c_t}{(1+r)^{t-S}} = b + \sum_{t=S}^{T} \frac{w\theta(1+z_j)}{(1+r)^{t-S}}$$

Combining this with the Euler equation, the optimal consumption path is given by:

$$c_S = \frac{1}{\Phi_S} \left(b + w(1+z_j)\theta\Phi_r^o\right)$$

$$c_t = ((1+r)\beta)^{\frac{t-S}{\sigma}} c_S, \quad t \in \{S, \dots, T\}$$

where Φ_S and Φ_r^o are given by the following expressions:

$$\boxed{\Phi_S = \frac{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{T-S+1}{\sigma}}}{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{1}{\sigma}}}} \qquad \boxed{\Phi_r^o = \frac{1 - \left(\frac{1}{1+r}\right)^{T-S+1}}{1 - \left(\frac{1}{1+r}\right)}}$$

The present value utility at time S of this consumption path is given by:

$$\sum_{t=S}^{T} \beta^{t-S} u(c_t) = \Phi_S u(c_S)$$

Note that c_S is determined for every given savings b carried from the college period, so without solving the problem for periods $\{0, \ldots, S-1\}$, it will not be completely pinned down. Now, we solve for the households' problem during periods $0, \ldots, S-1$. Given that during college, there exist exogenous borrowing constraints given by $\bar{A}$, there are two cases: a) individuals are unconstrained, and b) individuals are constrained. The unconstrained solution of the problem in periods $\{0, \ldots, S-1\}$ yields:

$$c_0\Phi_0 + (P_h h + P_l l)\Phi_r^y + \frac{a}{(1+r)^S} = w\theta\Phi_r^y(1-l)(1-h) + b$$

$$c_t = ((1+r)\beta)^{\frac{t}{\sigma}} c_0, \quad t \in \{1, \ldots, S-1\}$$

where b is the initial wealth of individuals, and Φ_0, Φ_r^y are given by:

$$\boxed{\Phi_0 = \frac{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{S}{\sigma}}}{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{1}{\sigma}}}} \qquad \boxed{\Phi_r^y = \frac{1 - \left(\frac{1}{1+r}\right)^S}{1 - \left(\frac{1}{1+r}\right)}}$$

Utility in period 0 is given by

$$\sum_{t=0}^{S} \beta^t u(c_t) = \Phi_0 u(c_0)$$

Note that now, the problem can be perfectly embedded in the two-period model described in Section 3. Households solve the following two-period problem:

$$\max_{c_0, c_S} \quad u(c_0) + \tilde{\beta} u(c_S), \quad s.t.$$

$$c_s\Phi_S = a + w\theta(1+z_j)\Phi_r^o$$

$$c_0\Phi_0 + (P_h h + P_l l)\Phi_r^y + \frac{a}{(1+r)^S} = w\theta\Phi_r^y(1-l)(1-h) + b$$

$$a \geq -\bar{A}$$

where:

$$\boxed{\tilde{\beta} = \frac{\beta^S \Phi_S}{\Phi_0}}$$

These two budget constraints can be rewritten as a single lifetime budget constraint:

$$c_0\Phi_0 + (P_h h + P_l l)\Phi_r^y + \frac{c_s\Phi_s}{(1+r)^S} = w\theta\Phi_r^y(1-l)(1-h) + \frac{w\theta(1+z_j)\Phi_r^o}{(1+r)^S} + b$$

The unconstrained consumptions are given by:

$$c_n = \frac{(\beta(1+r))^{(-S/\sigma)}\left[w\theta\left(\frac{\Phi_r^o + (1+r)^S\Phi_r^y}{\Phi_S}\right) + \frac{b(1+r)^S}{\Phi_S}\right]}{1 + \frac{(\beta(1+r))^{(-S/\sigma)}\Phi_0(1+r)^S}{\Phi_S}}$$

$$c_h = \frac{(\beta(1+r))^{(-S/\sigma)}\left[w\theta(1+z_h)\frac{\Phi_r^o}{\Phi_S} + \frac{b(1+r)^S}{\Phi_S} - \frac{P_h\Phi_r^y(1+r)^S}{\Phi_S}\right]}{1 + \frac{(\beta(1+r))^{(-S/\sigma)}\Phi_0(1+r)^S}{\Phi_S}}$$

$$c_l = \frac{(\beta(1+r))^{(-S/\sigma)}\left[w\theta(1+z_l)\frac{\Phi_r^o}{\Phi_S} + \frac{b(1+r)^S}{\Phi_S} - \frac{P_l\Phi_r^y(1+r)^S}{\Phi_S}\right]}{1 + \frac{(\beta(1+r))^{(-S/\sigma)}\Phi_0(1+r)^S}{\Phi_S}}$$