Optimal Taxation and Informality

Preliminary and incomplete, please do not circulate.

Rodrigo Azuero[†] Juan Hernandez [‡] Daniel Wills[§]

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Abstract

Informality is a widespread phenomenon in developing economies with negative consequences on productivity and inequality. Several policies have been implemented to decrease informality such as reducing corporate tax rates for small businesses or reducing payroll taxes to promote formal employment. However, these policies introduce new sets of distortions, and it is not clear whether the reform is optimal. In this paper, we develop a theory of optimal taxation in an economy with an informal sector. We construct a novel dataset combining a census of formal and informal businesses in Peru, administrative records from tax authorities and the national household survey in the country, which allow us to get a unique characterization of the informal economy. In our data, agents respond to tax increases both with behavioral distortions and by not abiding with the rules. The former effect appears to be greater. Next, we construct a positive model that can replicate the main features of the data, and use it to derive optimal non-linear tax formulas (à la Mirrlees) in this context.

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[†]Inter-American Development Bank. razuero@iadb.org +1 (202) 623-1000

[‡]Inter-American Develpoment Bank. juanhe@iadb.org +1 (202) 623-2255

[§]Universidad de los Andes d-wills@uniandes.edu.co +57 (1) 339-4949

1 Introduction

Informality is defined as the set of economic activities that occur outside of an economy's regulatory framework. This is a widespread phenomenon in developing economies. Approximately 46.8% of non-agricultural employees in Latin America are informal (Gomez, 2016) and around 40% of its GDP is produced in the informal sector. In Sub-Saharan Africa, 80% of the labor force is employed in the informal sector, which contributes to 55% of the GDP.

The high prevalence of informality has various negative consequences. Informality is usually associated with lack of protection on the worker side, and lack of compliance on the firm side. Governments cannot tax the informal sector, limiting the extent to which taxes and spending can be used as a tool for redistribution. Additionally, informality is thought to hinder productivity as firms and individuals deviate from optimal behaviors in order to avoid the scope of the government radar.

The negative consequences of informality brought the topic to the center of the academic and policy debate in developing countries (Perry, 2007). Poorly designed tax systems and burdensome regulations have been pointed as a potential cause of the high levels of informality (Levy, 2010). Naturally, the proposed fixes also point at reforms of the tax code. A popular policy involves introducing special tax regimes for small firms or other populations more prone to become informal. Another approach is to reduce payroll taxes and replace the foregone income by increasing other taxes. However, both measures introduce new distortions: when governments introduce size-dependent policies they create distortions leading to misallocation and with potentially large negative effects on productivity, and increasing corporate taxes distorts on organizational form and long-run capital accumulation, increasing labor income taxes lessens labor supply.

The discussion above suggests that we need a framework to characterize the optimal mix of different tax distortions in an economy where informality is a choice. We propose such a framework and characterize optimal, unrestricted, tax functions for business and for individual income. The model captures the basic choice between working and starting a firm, in a context where it is very hard for the government to observe small firm activity. How to fund a country's fiscal needs in an optimal way is one of the most studied questions in economics (Ramsey, 1927; Mirrlees, 1971; Stantcheva, 2017). However, the phenomenon of informality has not been taken into account in answering this question. In this paper we fill the gap by developing a theory of optimal taxation in an economy with an informal sector that captures the main features of this phenomenon.

An important challenge of any work related to informality is that of having good sources of information. By definition, the informal sector does not show up in administrative data and has to be measured by survey or census data. At the same time, the quality of survey and census data on income and tax payments is known to be limited, and the best source are the administrative records. In this paper, we combine novel sources of information for the informal sector in Peru to address the above challenges. We use data from the Economic Census of Peru, a unique dataset including financial and operational information of all establishments in the country, be it formal or informal. We also use aggregate administrative tax records provided from the national tax authorities to identify some features of the formal sector. Finally, we use the national household survey, ENAHO[1] to have an adequate description of the formal and informal labor force. To the best of our knowledge, in the context of developing countries, the only dataset comparable to the Economic Census of Peru is the Economic Census of Mexico[2], but Mexico does not make tax data available. By combining the economic census of Peru, together with administrative records from the tax authorities and the household survey of Peru, we obtain a unique data that allows us to get a detailed characterization of the formal and informal sectors in Peru.

In line with existing evidence for other developing countries, we find that informal employees are more prevalent in small firms, they earn less than formal ones, and tax evasion is decreasing in firm size. We also find kinks in the administrative records at the points where the law introduces discontinuities. The kinks can arise because the tax code distorts firms decisions, or because they lie when filing taxes. The comparison of tax records with census data allows us to identify behavioral distortions from misreporting separately. We do not find kins at the points of discontinuous tax treatment in the data reported only for statistical purposes, suggesting that misreporting plays an important role to explain those kinks.

We proceed in two steps. We first develop a positive model for Peru.

In the positive model, individuals chose to become entrepreneurs or to work for a wage in either, the formal or the informal sector. This allows accounting for the heterogeneity in occupational choice among informal workers observed in the data. Entrepreneurs maximize profits producing output with formal and informal employees. They can only pay payroll taxes on formal workers but face an increasing marginal cost on informal workers reflecting the fact that the more informal workers they hire, the more likely they are to be detected and the higher the expected fine from the authorities (As in Meghir, Narita, and Robin (2015) and Ulyssea (2017)). Entrepreneurs set up firms having to pay taxes on corporate profits -if big enough- but might choose to misreport them taking into account that larger deviations from the real profits are harder to justify and generate an expected penalty. Workers chose how many hours they provide in the formal and the informal labor market and pay income taxes.

The endogenous worker-entrepreneurial decision amplifies the effects of high payroll taxes combined with low (zero) corporate taxes for small firms. High payroll taxes induce workers to become entrepreneurs, and low corporate taxes for small firms provide incentives for low-scale operations. In turn, hiring informal workers is cheaper for small firms as the probability of detection is low.

We then solve for the allocation that maximizes a social welfare function reflecting preferences for redistribution. The planner can choose any arbitrary tax system, but cannot perfectly observe all

[1] In Spanish, "Encuesta Nacional de Hogares".

[2] Although El Salvador has also implemented an economic census, it excludes establishments with less than five workers. As we show in the description of this data, approximately 95% of businesses have fewer than five employees, which largely limits the analysis that can be done with such dataset.

economic activity. Specifically, the planner cannot observe informal markets. The planner chooses the optimal tax system subject to the observability restrictions, trading off efficiency and redistribution motives.

As occupational choice results in potentially different levels of skill for the same individual, the mechanism design problem cannot be solved using standard tools. We develop a method to simplify the problem and write it as an optimal control problem. This permits to deduce simple tax formulas from the optimality conditions of the problem.

The remainder of this paper is structured as follows. In Section 2 we describe the different datasets used in this paper and describe the main features of the informal and the formal sector. In Section 3 we develop a model of occupational choice incorporating the main features of the informal economy. After solving the (positive) model, we describe the problem of the benevolent social planner and characterize the optimal tax functions in section 4. The calibration of the model to the Peruvian data is the subject of Section 5, and we conclude in Section 6

2 Data

We use three sources of information: the 2007 Economic Census of Peru, the 2007 National Household Survey of Peru (ENAHO)[3], and aggregate administrative records from the tax administration (SUNAT)[4]. The Economic Census of Peru collects information from all establishments, formal or informal, operating in the year 2007 in Peru. A total of 940,336 establishments were surveyed in the census covering all economic sectors except for agriculture, public administration and defense, and economic activities that are not performed in fixed establishments. The information collected includes taxes payed, price and quantities of the main products and services sold, intermediate purchases, wages payed, financial statements, and use of technology, among others.

ENAHO is a standard household survey run by the national statistics department of Peru (INEI)[5]. It is run on a monthly basis on the 24 departments of Peru, including the Lima metropolitan region, and includes information about education, employment, income, expenses, and demographic composition of the household. A total of 22,640 households were surveyed in 2008 No es 2007 o 2006? including 8,816 rural and 13,824 urban households. The ENAHO survey is representative at the department (regional) level. Every year, approximately one third of the households are surveyed again to generate the panel sample of the survey. To have a comparable sample we limit our analysis to Lima, the capital and largest city of Peru and the only city for which there is a representative sample in the ENAHO. We also remove establishments with profits beyond the top 1%.

We obtain aggregate administrative records from the national tax authority in Peru (SUNAT).

[3]"Encuesta Nacional de Hogares" in spanish.

[4]In spanish: "Superintendencia Nacional de Aduanas y Administracion Tributaria".

[5]"Instituto Nacional de Estadística e Informática" in spanish.

Given the source of information, data from SUNAT is informative exclusively of formal businesses that report to the tax authorities. This information includes distribution of monthly sales for all establishments, profits, number of workers, workforce expenses.

2.1 Economic Census (2007)

There are 342,374 establishments in the economic census that operate in Lima. The distribution of such establishments by sector of economic activity is reported in Table 1. Commerce, hotels and restaurants and manufacturing encompass 80.95% of the establishments in the Census.

Table 1: Distribution of establisments by sector of activity

	N	Percentage
Administrative and support	4,139	1.76
Arts	1,586	0.67
Commerce	137,813	58.57
Construction	1,390	0.59
Education	5,174	2.20
Elecricity and gas	62	0.03
Financial sector	652	0.28
Fishing	798	0.34
Health	4,274	1.82
Hotels and restaurants	28,940	12.30
Manufacturing	23,716	10.08
Mining	153	0.07
Other services	14,621	6.21
Professional/Scientific	6,128	2.60
Real estate	775	0.33
Transportation and storage	4,822	2.05
Water treatment/provision	235	0.10

Information on financial balances, sales, and general operation, are only available for establishments that were fully operational in the year 2007. For this reason, although 342,374 establishments were included in the census, the questionnaire about financial information was answered by 235,278. We report some statistics for these establishments in Table 2[6] after removing the top 1% in profits.

We note that most establishments are young as the average age is 7.2 years and less than 25% of establishments have been operating for more than a decade. The median establishment has annual profits of $11,327 USD and the average amount of taxes payed in the form of corporate income is $605.6 USD. This corresponds to an average payment of 5% of profits in the form of corporate profit

[6]Monetary variables are reported in $USD considering an exchange rate of 0.315 USD/ PEN (sol).

tax. It is also important to note that less than 25% of establishments are actually paying some form of corporate income tax. The value of all assets included in the operation is, on average, four times the level of profits. The average establishment size, in terms of number of employees, is 4.35 and less than 25% of them have more than 3 workers.

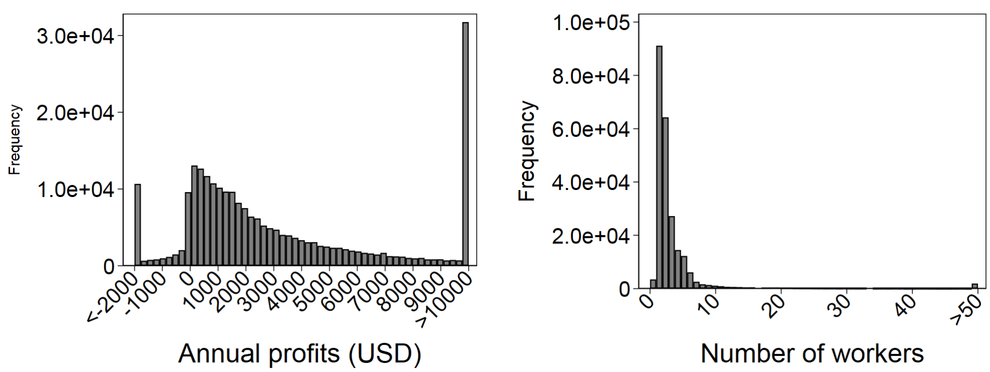
Table 2: Descriptive Statistics

	Mean	Median	Std. Dev	P-25	P-75	N
Age	7.23	4.00	7.92	2.00	10.00	235,278
Profits (USD)	11,327.36	1,952.37	540,869.76	585.27	5,142.93	235,278
Corporate Income Tax (USD)	605.62	0.00	4,107.30	0.00	0.00	235,278
Assets (USD)	45,901.20	315.00	1,635,276.70	0.00	1,575.00	235,278
Workers	4.35	2.00	34.93	1.00	3.00	235,278

Note: monetary variables are reported in $USD considering an exchange rate of 0.315 USD/ PEN (sol).

In Figure 1 we explore further the size distribution of firms in terms of profits and number of workers. Most very few establishments report profits over $10,000, and most of them employ between one and two workers.

Figure 1: Size Distribution of firms



Although small businesses are prevalent, these business employ a small share of productive resources, and explain a small portion of the overall taxes payed in and of the aggregate value added in the economy. In Table 3 we note that establishments with fewer than five employees represent 90% of the distribution but only employ 41% of the workers, utilize 14% of the total physical capital being used in the data, contribute 21% to the total value added of establishments in the census and pay 24% of all taxes. The establishments employing more than fifty workers represent 1% of the total distribution but they employ 34% of workers, use 53% of the total physical capital, explain 48% of the total value added and are responsible for the 32% of total tax payments of all establishments.

Table 3: Share of establishments/workers/capital/VA/taxes by firm size

Employees	Establishments	Employees	Capital	Value Added	Taxes
$[0-5]$	0.90	0.41	0.14	0.21	0.24
$[6-10]$	0.05	0.09	0.07	0.08	0.10
$[11-50]$	0.03	0.16	0.27	0.23	0.34
[50+]	0.01	0.34	0.53	0.48	0.32

2.2 ENAHO (2007) Household Survey

The ENAHO survey of 2007 contains information for 95,469 individuals, out of which 11,608 live in Lima. The size of the economically active population, composed of those who are working or who are looking for a job, is of 6,050 individuals. Out of those, 5.97% are unemployed. We present some descriptive statistics of those who are working in Table 4.

Table 4: Descriptive Statistics (ENAHO)

	Mean	Median	Std. Dev	P-25	P-75	N
Age	37.55	36.00	14.46	26.00	48.00	6,004
Monthly income	216.32	160.11	244.45	58.60	278.03	6,004
Schooling (years)	10.79	11.00	3.86	9.00	14.00	6,004
Men	0.54	1.00	0.50	0.00	1.00	6,004
Contribute to Social Security	0.20	0.00	0.40	0.00	0.00	6,004

Note: monetary variables are reported in $USD considering an exchange rate of 0.315 USD/ PEN (sol).

Individuals are on average 37 years old, their monthly income is the equivalent of $216.32 USD and have 10.8 years of schooling. 54% of them are men and only 20% report to contribute to social security, which is often used as an indicator of informality. In Table 5 we report the distribution of sectors among the workers in the sample. Comparing the distribution of the workforce across sectors of economic activity with that of establishments reported in Table 1, we note that commerce, restaurants and hotels, manufacturing, and services, are the most prevalent sectors.

Table 5: Distribution of sector of activity (ENAHO)

	N	Percentage
Commerce, restarutans, hotels	1,654	29.32
Construction	335	5.94
Elecricity, gas, water	11	0.19
Financial sector	60	1.06
Fishing	438	7.76
Manufacturing	865	15.33
Mining	29	0.51
Services	1,735	30.75
Transportation and storage	515	9.13

The definition of informality that we follow in this work is that of economic activities that are legal but that are not regulated or taxed by the corresponding authorities. As such, we define an employee to be in an informal labor relationship if she does not have a written contract guaranteeing the benefits and responsibilities established in the labor code. For self-employed and employers, ENAHO asks the question of whether or not their main occupation is in the informal sector or not. Non-remunerated workers are considered by definition as informal workers and we consider workers who report "other" occupational category to be informal if they have no contract. We report the distribution of informal workers in each occupational category according to this definition in Table 6.

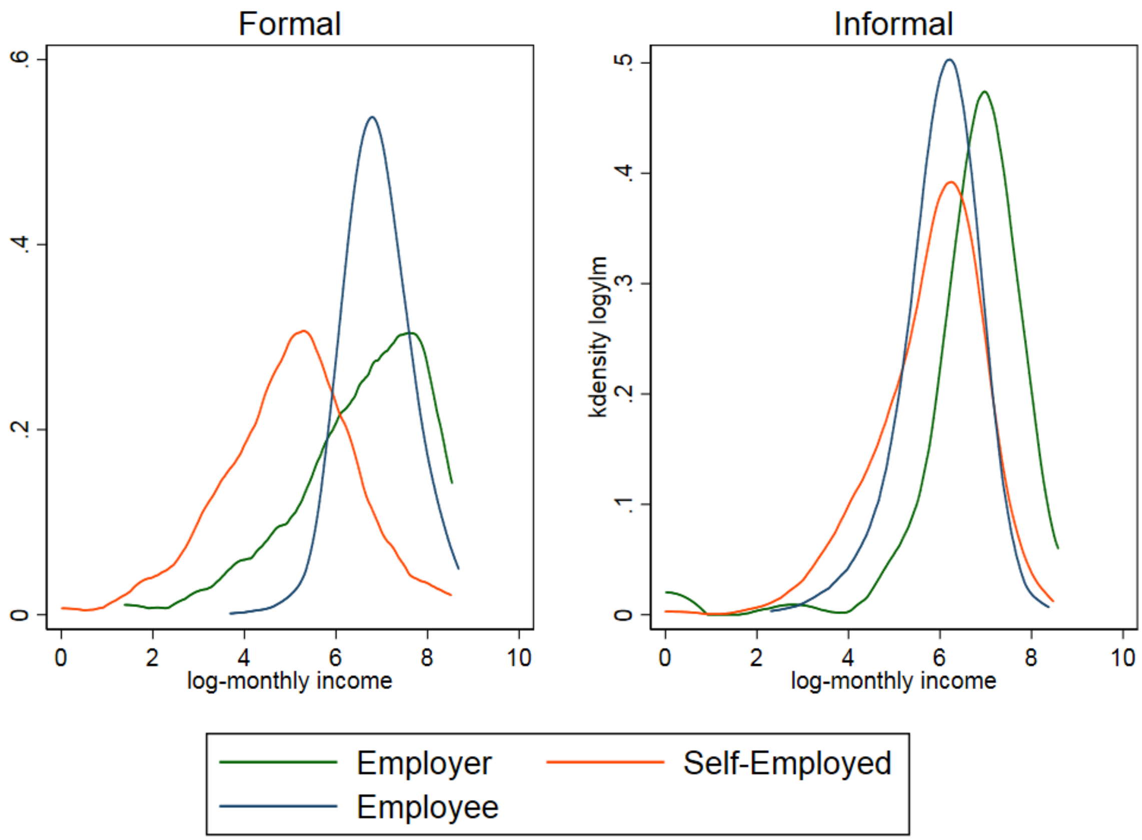
Table 6: Distribution of occupational categories and informality (ENAHO)

	Number of individuals	% in labor force	% who are informal
Employee	3,354	59.38	53.46
Employer	328	5.81	75.91
Non-remunerated	356	6.30	100.00
Other	8	0.14	100.00
Self-employed	1,602	28.36	92.38
Total	5648	100.00	68.76

We observe that most individuals are employees and approximately half of them are informal. Self-employed is the next category in terms of proportion of individuals working in such a way, corresponding to 28% of which 92% are informal. Out of the 5.86% of individuals who are employers, 76% are informal and individuals who report "other" occupational category are informal.

We report the distribution of wages in Figure 2 and Tables 7 and 8. Within formal workers, employers are the best remunerated, followed by employees. Self-employed workers in the formal sector earn about one fourth of what employers earn. For informal workers, employers are still the highest payed but employees and self-employed earn about the same. It is important to recall that the number of formal self-employed workers is relatively small.

Figure 2: Distribution of earnings



Note: density estimates using Epanechnikov kernel, bandwidth=0.5.

Table 7: Monthly earnings - Formal workers

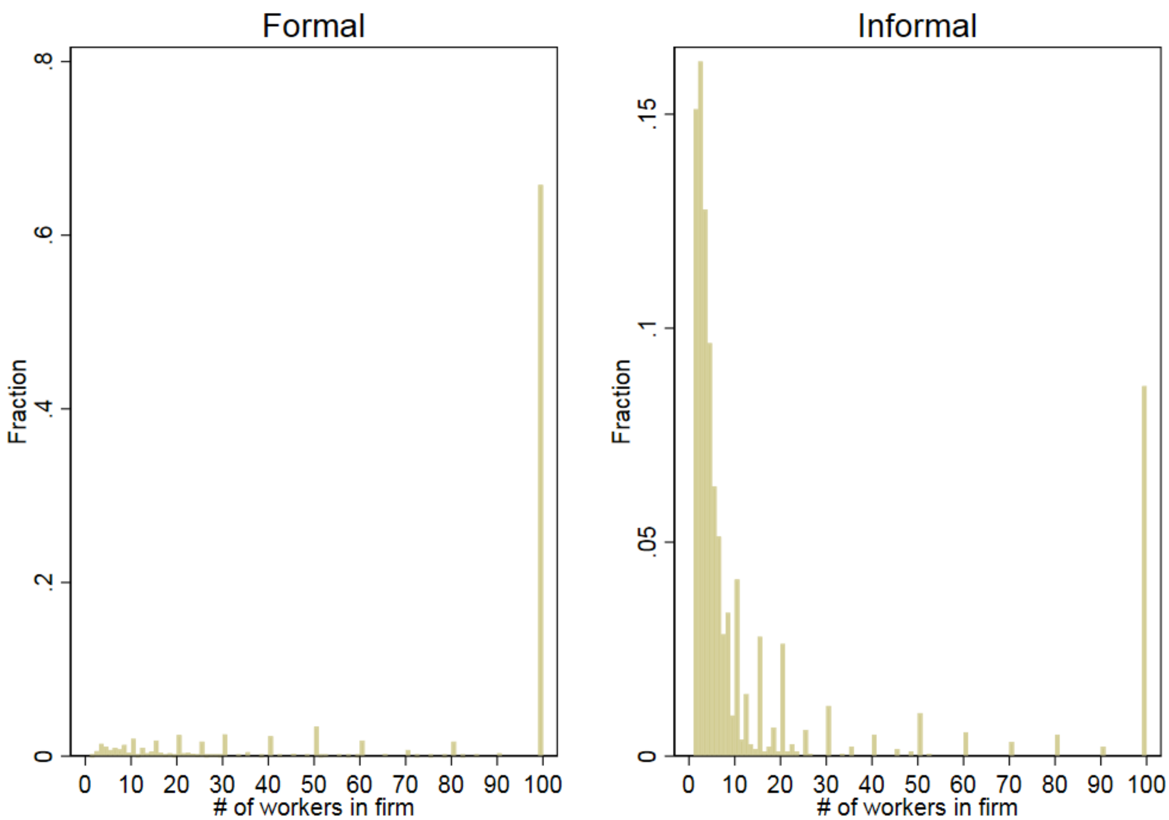
	Employees	Employers	Self-employed
Mean	388.01	443.33	115.62
SD	297.76	426.51	213.70

Table 8: Monthly earnings - Informal workers

	Employees	Employers	Self-employed
Mean	168.66	391.79	169.88
SD	131.80	316.68	173.19

Finally, we note that among employees, informality is correlated with firm size. Most informal workers are concentrated in small firms whereas the distribution is more spread for formal workers. In figure 3 we show how informal and formal workers are distributed across firm size.

Figure 3: Distribution of informal workers and establishment size

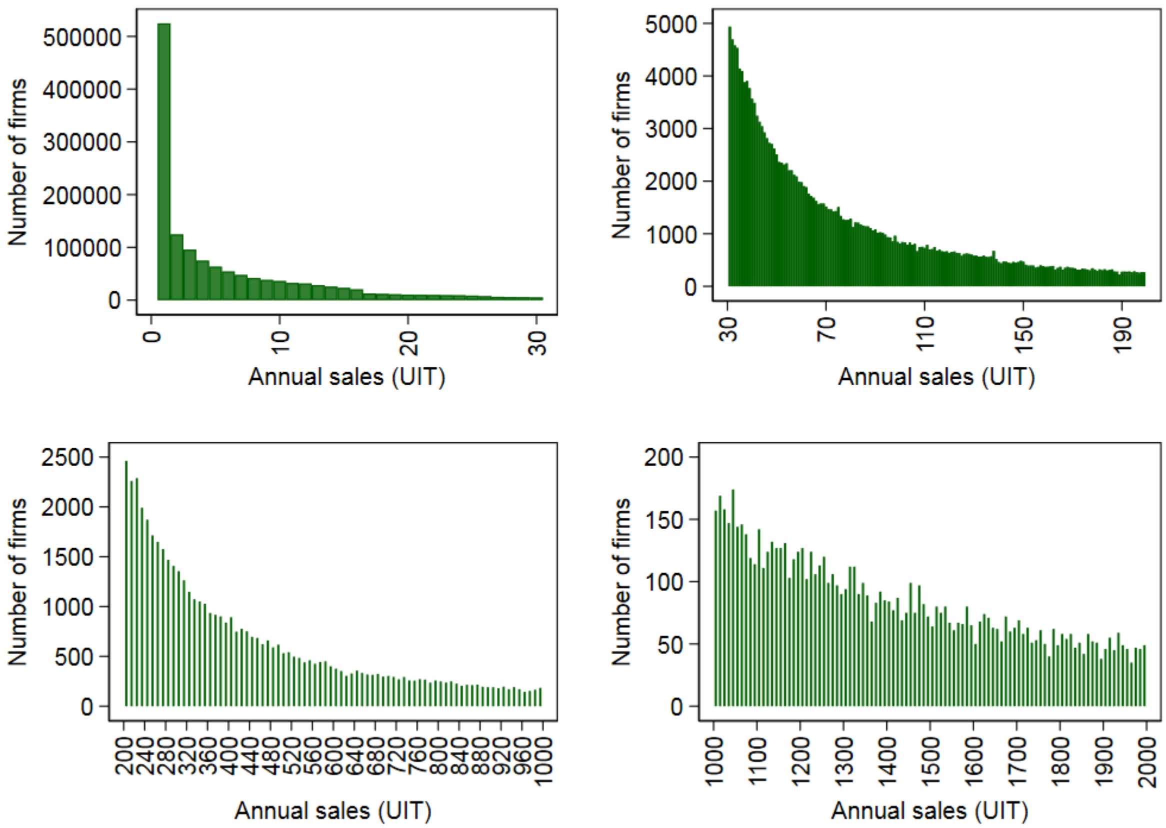


2.3 SUNAT dataset

The information provided by SUNAT includes distribution of monthly sales for all establishments, profits, number of workers, and workforce expenditure. In Figure 4 we report the distribution of firms by the volume of their annual sales in 2014, in UIT units[7].

[7]SUNAT uses UIT units as a measure of reference. In 2014 1 UIT=3,800 Sol=$1,177 USD.

Figure 4: Distribution of firms by annual sales

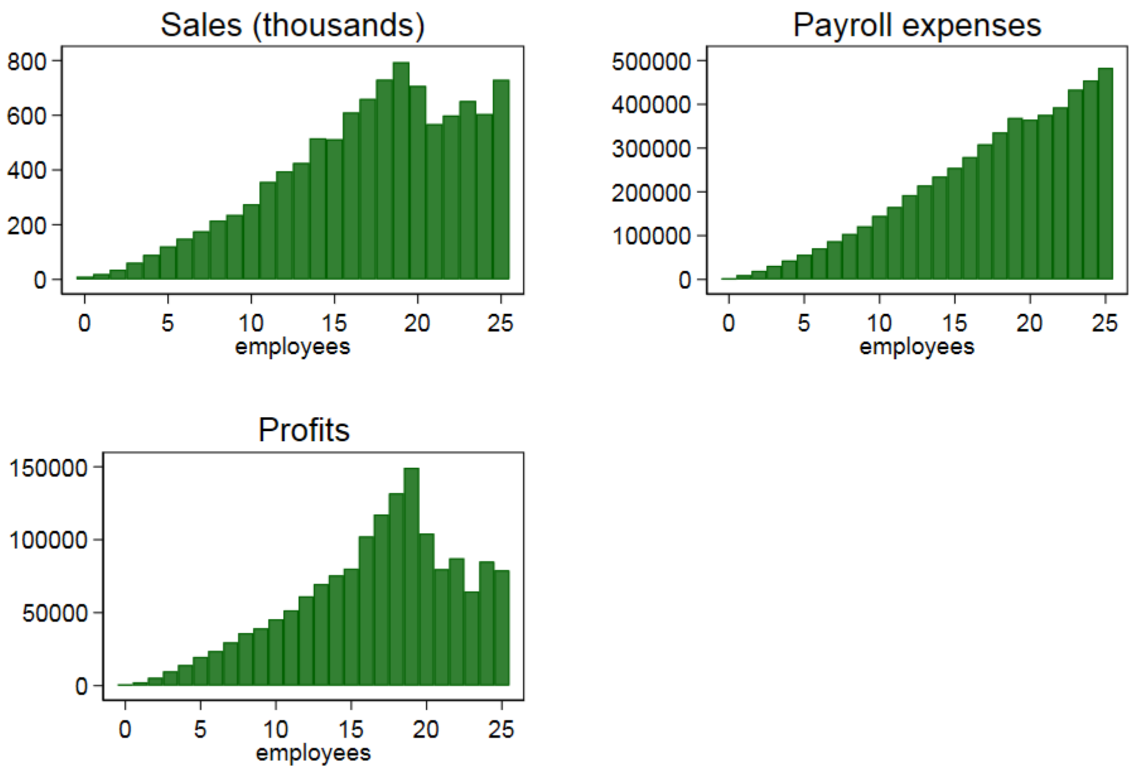


Note: 1 UIT=3,800 Sol=$1.177 USD.

This distribution only includes formal firms. However, even in the formal sector, most firms are small: 524,661 firms report sales of less than $1,177 USD, out of a total of 1,647,529.

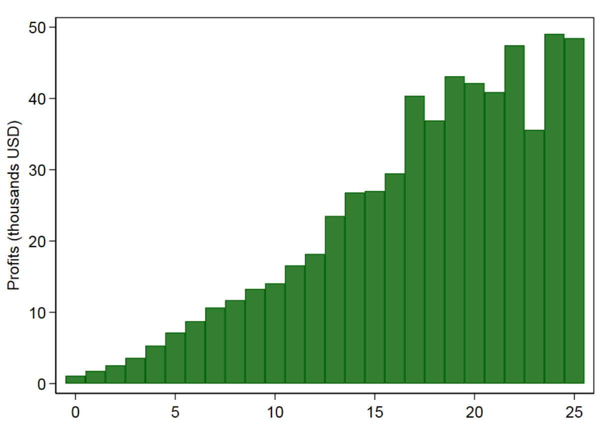
In Figure 5 we report median sales, payroll expenses, and profits, for 2014, depending on the number of employees a firm reports to have. We observe an increasing trend in the three series but we notice a discontinuity in sales and profits around twenty workers. In Peru, firms with more than twenty workers are subject to a regime in which they have to distribute a proportion of their profits with their workers. The proportion depends on the economic sector of activity for each firm. For firms in communications and manufacturing this figure corresponds to 10% of their after-tax profits, 8% for mining, services, restaurants and hotels and 5% for the remaining sectors. We argue that such regulation generates an incentive for firms to misreport their profits might explain the discontinuity observed in both, sales and profits to avoid their tax burden as there are no technological reasons why we should observe such a pattern in these series.

Figure 5: Median annual sales, profits, and payroll (USD)



In Figure 6 we report the median profits for establishments according to the number of employees. Note that the discontinuity observed in Figure 5 is not present in the reports of the Economic Census.

Figure 6: Median profits (USD) in the Economic Census



3 Model

We analyze a static economy that captures the basic entrepreneurial choice model and analyzes optimal taxes in such a setting. We study distortions to the entry margin into entrepreneurship as well as taxes on payroll, labor income and corporate income.

3.1 Primitives

The economy consists of a continuum of individuals and a government.

Individuals: agents in this economy can do one of two things, become an entrepreneur or work for a wage. If they decide to work for a wage they also need to decide the distribution of time between the formal and informal sector. Entrepreneurs (firms) can choose to hire formal or informal workers, for which they do not pay payroll taxes. Additionally, they can choose to misreport their profits to avoid taxes. Individuals are heterogeneous with respect to how productive they are in each activity. In particular, each individual is identified by a pair $\theta = (\theta_w, \theta_e)$ where θ_w denotes productivity in the labor market and θ_e is entrepreneurial productivity and in each sector $s \in \{w, e\}$ we assume $\theta_s \in \left[\underline{\theta_s}, \overline{\theta_s}\right]$.

Workers: Upon becoming a worker of skill θ_w, an individual decides how much time to allocate to informal work l_i and how much time to spend in the formal labor market l_f. w_s denotes the wage rate in the sector $s \in \{i, f\}$. A worker of skill θ_w provides $l_s\theta_w$ effective units of work in sector s. Income in each sector is given by $\theta_w l_s w_s$. Supplying labour hours, independently of their nature, generates a disutility of $V(l) = \frac{\chi}{1+\psi} l^{1+\psi}$ to the worker, where $l = l_f + l_i$.

Workers pay taxes according to the function $T_l(w_f\theta_w l_f)$, which can be negative to denote a transfer from the government. The transfer function $T_l(\cdot)$ depends exclusively on the income from the formal market. We assume that income from the informal market is not observed by the tax authority. For this reason, individuals might decide to provide labor supply in the informal labor market to manipulate the transfer function in their benefit. However, participating in the informal market is increasingly costly and this is denoted by a utility penalty $k_l\big(\theta_w l_i(\theta_w)\big) = \frac{\kappa}{1+\rho}\left(\theta_w l_i(\theta_w)\right)^{1+\rho}$. This penalty incorporates the fact that it is costly to supply labor in the informal sector either because it becomes increasingly harder to hide large amounts of income or because individuals are excluded from the benefits of the formal labor market such as access to health insurance, unemployment benefits, and pensions, among others.

Entrepreneurs: Entrepreneurs are characterized by skill θ_e. They can produce $y = \theta_e q(n) = \theta_e n^{\alpha}$, where n is the total effective labor input. Labor can be hired from the formal and informal markets. Moreover, the entrepreneur has an endowment of effective labor ϵ that is supplied inelastically in her firm. All sources of effective work hours are perfect substitutes in the production function: $n = n_i + n_f + \epsilon$.

Entrepreneurs pay payroll taxes on the value of their formal payroll. Payroll taxes are given by a function $T_n(n_f w_f)$. Informal hires are not observed by the authorities, and hence does not comply with regulations. However, entrepreneurs face a cost $k_n(n_i)$ when hiring informal workers. In line with Ulyssea (2017) and Meghir et al. (2015), we interpret this cost to be an expected penalty and we assume it to be increasing and convex. We also assume that monitoring informal activity is costly for the government. To simplify matters, we will start by assuming that the enforcement agency breaks even: the cost of monitoring is equal to the revenue raised by the fines and forfeits. This assumption will be relaxed later on. We will use the parametric function $k_n(n_i) = \frac{\delta}{1+\gamma} n_i^{1+\gamma}$. In addition to payroll taxes, entrepreneurs face a corporate profit tax (T_c), payed on the operating profits of firms. As for payroll taxes, we allow the function $T_c(\cdot)$ to take arbitrary forms, as long as it only depends on operating profits. Motivated by the empirical evidence provided in section 2, we allow entrepreneurs to under-report operating profits by an amount (z). Underreporting profits is also costly and we assume an increasing and convex cost denoted by $k_c(z) = \frac{\beta}{1+\sigma} z^{1+\sigma}$.

We denote by $i \in \{0, 1\}$, an individual's decision about entry into entrepreneurship where $i = 1$ represents entry into entrepreneurship.

Government: The role of the government is to raise taxes in order to cover its expenses G and pay for any transfers implied by the tax scheme, trading off efficiency and redistribution motives, and subject to information frictions.

Allocations: An allocation in this economy is described by specifying consumption, as well as entrepreneurial choice, hours worked in case of becoming a worker, and formal and informal labor hired by entrepreneurs given by:

$$\{c(\theta), i(\theta), l_f(\theta), l_i(\theta), n_f(\theta), n_i(\theta), z(\theta)\}_{\theta \in \Theta}. \tag{1}$$

An allocation is said to be feasible if it satisfies:

$$\begin{aligned} G + \int_\Theta c(\theta)\, dF(\theta) = \int_\Theta \Big\{ \Big[\theta_e q\big(n(\theta_e)\big) - k_n\big(n_i(\theta_e)\big) - k_c\big(z(\theta_e)\big)\Big] i(\theta) \\ - k_l\big(\theta_w l_i(\theta_w)\big)\big(1 - i(\theta)\big) \Big\} dF(\theta) \end{aligned} \tag{2a}$$

$$\textstyle\int_\Theta n_f(\theta_e) i(\theta)\, dF(\theta) = \int_\Theta \theta_w l_f(\theta)(1 - i(\theta))\, dF(\theta) \tag{2b}$$

$$\textstyle\int_\Theta n_i(\theta_e) i(\theta)\, dF(\theta) = \int_\Theta \theta_w l_i(\theta)(1 - i(\theta))\, dF(\theta) \tag{2c}$$

The first equation states that all the output -net of the efficiency costs resulting from non-compliance- is consumed. The second equation is the formal labor market clearing condition. And the third equation is the informal labor market clearing condition. Notice the composition of workers is irrelevant for production by entrepreneurs. In other words, entrepreneurs are assumed to be hiring a representative population of workers, and workers choose to take formal or informal jobs.

3.2 Equilibrium with Taxes

In this section we describe the competitive equilibrium, taking tax functions as given. Later in section 4, we will solve for the optimal tax functions given a social welfare function. There are only two commodities in the economy, namely consumption good and units of effective labor. We use w_f to denote the price of an effective unit of formal labor in terms of consumption good, and similarly w_i denotes the relative price of informal labor.

Entrepreneurs

We define the operating profits of an entrepreneurial firm, $\pi(\theta_e, n_i, n)$, as production production $\theta_e n^\alpha$ net of payroll $w_i n_i + w_f[n - n_i - \epsilon]$ and payroll taxes $T_n(w_f[n - n_i - \epsilon])$.

$$\pi(\theta_e, n_i, n) = \theta_e n^\alpha - w_i n_i - w_f[n - n_i - \epsilon] - T_n\big(w_f[n - n_i - \epsilon]\big).$$

Entrepreneurs choose the number of formal workers to hire n_f the number of workers to hire n, the number of informal workers to hire n_i, and how much of its profits to hide, in order to maximize her benefits.

The benefits of an entrepreneur of ability θ_e are described in equation 3,

$$\begin{aligned} u_e(\theta_e) = \max_{n, n_i, z} \, & \theta_e n^\alpha - w_i n_i - w_f(n - n_i - \epsilon) - T_n((n - n_i - \epsilon) w_f) \\ & - T_c\left(\theta_e n^\alpha - w_i n_i - w_f(n - n_i - \epsilon) - T_n((n - n_i - \epsilon) w_f) - \frac{\delta}{1+\gamma} n_i^{1+\gamma} - z\right) \\ & - \frac{\delta}{1+\gamma} n_i^{1+\gamma} - \frac{\beta}{1+\sigma} z^{1+\sigma}. \end{aligned} \tag{3}$$

The first line of equation 3 displays operating profits. The second line shows corporate income taxes. Note that the base of the tax is the operating profit net of under-reporting and fines for informality[8]. The third line, represents the costs of not complying with regulations. The term $\frac{\delta}{1+\gamma} n_i^{1+\gamma}$ is the cost of deviating workers to the informal sector, and the term $\frac{\beta}{1+\sigma} z^{1+\sigma}$ is the cost of under-reporting profits.

The optimality conditions characterizing the solution of problem 3 are the following,

$$\big(\alpha \theta_e n^{\alpha-1} - w_f\big(1 + T_n'((n - n_i - \epsilon) w_f)\big)\big)\, \big(1 - T_c'(\pi(\theta_e, n_i, (n - n_i - \epsilon)) - z)\big) \leq 0, \tag{4}$$

$$\big(-w_i + w_f(1 + T_n'((n - n_i - \epsilon) w_f)) - \delta n_i^\gamma\big)\, \big(1 - T_c'(\pi(\theta_e, n_i, (n - n_i - \epsilon)) - z)\big) \leq 0, \tag{5}$$

$$T_c'(\pi(\theta_e, n_i, (n - n_i - \epsilon)) - z) = \beta z^\sigma \tag{6}$$

$$n = \epsilon, \; n_i = 0, \text{ if } \alpha \theta_e \epsilon^{\alpha-1} < \min\{w_f\big(1 + T_n'(0)\big), w_i\}. \tag{7}$$

Equation 4 equates the marginal cost of an effective hour of labor input with its marginal benefit. Equation 5 implies that the benefit of hiring a worker informally instead of formally -that is, the net savings in payroll taxes-, is equal to the marginal increase in the expected penalty of hiring

[8] For mathematical convenience we assume that fines for informality are tax-deductible but fines for tax evasion are not. All the results are robust to relaxing this assumption.

informal workers. Equation 6 says that optimal under-reporting occurs when the marginal savings in corporate taxes are equal to the marginal change in the corresponding expected penalty. Falta explicar ecuación 7

To gain some intuition about firm behavior, we consider the special (but empirically common) case of constant marginal tax rates. If we assume that corporate taxes are not confiscatory ($T_c' < 1$), the firm size is given by,

$$n = \left(\frac{\alpha \theta_e}{w_f (1 + T_n')} \right)^{\frac{1}{1-\alpha}}. \tag{8}$$

That is, firm size is increasing in managerial ability. Also notice that in this special case,

$$n_i = \left(\frac{w_f (1 + T_n'(\cdot)) - w_i}{\delta} \right)^{\frac{1}{\gamma}}. \tag{9}$$

With flat taxes, firms would hire a number of informal workers that is constant in managerial ability. That number is zero when $w_f\big(1 + T_n'\big) = w_i$.

The two observations above, imply that the fraction of informal workers is decreasing in firms size, and that very small firms do not hire formal workers. This is in accordance with the empirical evidence described in section 2.

Last, when taxes are flat, evasion is given by,

$$z = \left(\frac{T_c'}{\beta}. \right)^{\frac{1}{\sigma}} \tag{10}$$

The equation above says that firms hide a constant amount of their profits, that would be zero in the absence of corporate taxes. As size and profits are increasing in ability, very small firms do not report any profits, consistent with the data from the Economic Census.

Workers

The workers' problem takes the following form,

$$u_w(\theta_w \mid w_f, w_i) = \max_{l, l_i} \theta_w \left(w_f (l - l_i) + w_i l_i \right) - \frac{\chi}{1+\psi} l^{1+\psi} - \frac{\kappa \left(\theta_w l_i \right)^{1+\rho}}{1+\rho} - T_l \left(\theta_w w_f (l - l_i) \right). \tag{11}$$

subject to $l_s \geq 0$ for $s = i, f$. We assume non-confiscatory taxes. That is, $T_l' \left(\theta_w w_f (l - l_i) \right) < 1$ at every point. A worker will supply a positive amount of labor in the informal market $l_i \in [0, 1]$ up to the point where the optimality condition holds:

$$\theta_w (w_i - w_f (1 - T_l' (\theta_w w_f (l - l_i)))) - \kappa \theta_w^{1+\rho} l_i^{\rho} = 0. \tag{12}$$

Equation 12 is the standard condition equalizing the marginal benefit of working, which in our quasi-linear environment is labor market income from the informal sector, with the marginal dis-utility of working.

Similarly, worker's supply satisfies

$$\theta_w w_f\big(1 - T_l'(\theta_w w_f(l - l_i))\big) - \chi l^{\psi} = 0. \tag{13}$$

For the case of constant marginal tax-transfer rates[9], equation 12 implies that hours worked are monotonically increasing in ability.

We obtain:

$$\frac{l_i^{\rho}}{l^{\psi}} = \frac{\chi}{\kappa \theta_w^{1+\rho}} \left(\frac{w_i}{w_f\big(1 - T_l'\big)} - 1 \right).$$

If the additional disutility of providing informal labor depends only on hours, as opposed to effective hours, the $\theta_w^{1+\rho}$ term in the denominator above disappears. Without this term we can still make the ratio $\frac{l_i}{l_f + l_i}$ to be decreasing on θ_w by choosing a very large ρ relative to ψ, but l_i would be increasing on θ_w.

Definition of Equilibrium

An equilibrium with taxes consist of an allocation and wages w_f, w_i such that

- $i(\theta) = 1$ whenever $\Pi(\theta_e) > W(\theta_w)$ No se ha definido esto..
- If $i(\theta) = 1$, the allocation for θ solves problem 3, given taxes and prices.
- If $i(\theta) = 0$, the allocation for θ solves problem 11, given taxes and prices.
- The allocation is feasible.
- The government budget is balanced,

$$\textstyle\int_{\Theta} \left\{ \Big(T_c\big(\pi(\theta_e)\big) + T_n\big(w_f n_f(\theta_e)\big)\Big) i(\theta) + T_l\big(w_f \theta_w l_f(\theta)\big)\,(1 - i(\theta)) \right\} dF(\theta) = G. \tag{14}$$

4 Planner's Problem

In the discussion above, we introduced corporate, payroll and labor income taxes as arbitrary functions of profits, formal payroll and income from formal labor respectively. Our key assumption about the information structure is that individuals privately observe their productivity θ vector and also privately decide about working effort (in case they become a worker) or evasion and informal hiring (in case they become an entrepreneur). We assume that the choice to become an entrepreneur or a worker is observable to the planner and thus the taxation authority can tailor the tax code accordingly.

[9] Bhandari, Evans, Golosov, and Sargent (2013) show that a constant marginal tax-transfer function is a good approximation for the case of the United States.

As is standard in the optimal taxation literature, we will solve the dual problem. The planner will choose an allocation facing the same informational constraints as the tax authority in the decentralized equilibrium. The planner will choose the allocation that maximize some notion of social welfare, by taking into account the physical constraints and the incentive compatibility conditions associated with such allocations. Finally, an optimal tax policy will be backed out from the chosen allocations.

Implementable allocations

Recall an allocation in this economy is described by specifying consumption, as well as entrepreneurial choice, in case of becoming a worker hours worked in the formal and informal market, and formal and informal labor hired by entrepreneurs:

$$\left\{c(\theta), i(\theta), l(\theta), l_i(\theta), n(\theta), n_i(\theta), z(\theta)\right\}_{\theta\in\Theta}$$

To state the dual planner's problem, we need to restrict the available allocations the planner can choose from. In addition to the feasibility conditions, we call an allocation *implementable* if there exist payroll, corporate and personal income tax functions $T_n(\cdot), T_c(\cdot)$ and $T_l(\cdot)$ and wages w_f, w_i such that the allocation together with those tax functions and wages are a tax equilibrium.

The planner's proposed allocation constitutes a direct mechanism for the agent. For that mechanism to be incentive compatible, it requires that every agents weakly prefers the corresponding allocation assigned to his/her type θ over the allocations available *for other* types $\theta \in \Theta$. However the agent must keep in mind that when pretending to be a different type, he/she has to be consistent with the choices *observable* for the planner: effective hours in the formal labor market and *declared* sales. In addition, even when reporting their true type, agents should be maximizing utility with their unobservable actions as long as they are consistent with outcomes observable to the planner.

For example, in any mechanism that prescribes a type (θ_w, θ_e) agent to supply $\hat{l}_f$ hours, the his/her informal hour supply solve:

$$\check{u}_i(\theta, \hat{l}_f) = \max_{l_i} w_i\theta_w l_i - \frac{\chi}{1+\psi}\left(\hat{l}_f + l_i\right)^{1+\psi} - \kappa\frac{(\theta_w l_i)^{1+\rho}}{1+\rho}, \tag{15}$$

where we define $\check{l}_i(\theta, \hat{l}_f)$ to be the arg max of the problem described above in (15). This implies that any implementable mechanism should have $l_i(\theta) = \check{l}_i\big(\theta, \hat{l}_f(\theta)\big)$.

In addition, when an agent of type θ pretends to be of type θ' he/she must adjust his/her choices. If the planner assigned type θ' to be a worker, this is $i(\theta') = 0$, then the agent must provide $\frac{\theta'_w}{\theta_w} l_f(\theta')$ hours, to satisfy the planner's effective hours demand. However, as hours worked informally are not observable, the agent is free to chose any amount of informal hours, hence he/she provides $\check{l}_i\Big(\theta, \frac{\theta'_w}{\theta_w} l_f(\theta')\Big)$ hours of informal work.

On the other hand, if the planner assigned type θ' to be an entrepreneur, the agent is forced to use $n_f(\theta')$ hours of formal workers, but is free to hire any amount of informal hours as long as he/she *declares* the expected amount of sales by the planner, which is $\theta_e\big(n_f(\theta')+n_i(\theta')+\epsilon\big)^\alpha - z(\theta')$. Hence, if the choice of informal hours is $\check{n}_i$ the corresponding choice of profit hiding is:

$$\check{z}(\check{n}_i,\theta';\theta) = z(\theta') - y(\theta') + \theta_e\big(n_f(\theta') + \check{n}_i + \epsilon\big)^\alpha, \tag{16}$$

where $y(\theta') = \theta'\big(n_f(\theta') + n_i(\theta') + \epsilon\big)^\alpha$. Recall the agent always has access to the informal labor market. The problem for a type θ agent pretending to be an entrepreneur of type θ' is:

$$\check{\Pi}(\theta';\theta) = \max_{\check{n}_i} \theta\big(n_f(\theta') + \check{n}_i + \epsilon\big)^\alpha - w_i\check{n}_i - \frac{\delta\check{n}_i^{1+\gamma}}{1+\gamma} - \beta\frac{\check{z}(\check{n}_i,\theta';\theta)^{1+\sigma}}{1+\sigma}. \tag{17}$$

Notice that, as in the worker case, in any implementable direct mechanism that prescribes an entrepreneur of type θ_e to hire $n_f(\theta_e)$ formal hours must specify the level of informal hours that solves the problem described in (17) when $\theta' = \theta$.

A direct mechanism defines an utility allocation for each agent of type θ:

$$u(\theta) = c(\theta) - \big(1 - i(\theta)\big)\frac{\chi}{1+\psi}l(\theta)^{1+\psi}.$$

Hence an allocation is incentive compatible if $\forall\theta,\theta' \in \Theta$

$$u(\theta) \geq u(\theta') + \big(1 - i(\theta')\big)\left[\check{u}_i\left(\theta_w, \frac{\theta'_w}{\theta_w}l_f(\theta'_w)\right) - \check{u}_i\big(\theta'_w, l_f(\theta'_w)\big)\right] + i(\theta')\big[\check{\Pi}(\theta'_e,\theta_e) - \check{\Pi}(\theta'_e,\theta'_e)\big], \tag{18}$$

where the terms in brackets contain the change in consumption and leisure obtained from operating at a different scale and input mix from the planner's suggested one, net of working and compliance disutilities.

We assume that the government's objective is given by

$$\int_\Theta W\big(u(\theta)\big)f(\theta)d\theta, \tag{19}$$

where $W(\cdot)$ is an increasing and concave function, $u(\theta)$ is the utility of an individual of type θ defined above and $f(\cdot)$ is a function that captures the mass of people with productivity θ. A special case that gives us analytical tractability is the Rawlsian objective given by:

$$\min_{\theta\in\Theta} u(\theta).$$

Recall that incentive compatibility requires that the utility is increasing in θ_w so that,

$$\min_{\theta\in\Theta} u(\theta) = u_w(\underline{\theta_w})h(\underline{\theta_w}).$$

An allocation is said to be constrained efficient if it maximizes (19) while satisfying (18) and (2).

Simplifications. In order to simplify the optimization problem involving the efficient allocation, we make four observations: First, if two individuals have the same labor productivity and the allocations prescribes them that they become worker, incentive compatibility implies that they should receive the same utility. The same holds for entrepreneurs. It can also be inferred that all the workers with the same productivity and all entrepreneurs with the same productivity must have the same allocation. Thus we can define allocation in terms of the occupational choice $\{c_w(\theta_w), l_f(\theta_w), c_e(\theta_e), n_f(\theta_e), n_i(\theta_e), z(\theta_e), i(\theta)\}$ together with the utility profiles $\{U_e(\theta_e), U_w(\theta_w)\}$. Slightly abusing notation, we can write the incentive constraints as:

$$\begin{aligned} u_e(\theta_e) &\geq c_e(\theta_e') + \check{\Pi}(\theta_e', \theta_e) - \check{\Pi}(\theta_e', \theta_e'), \\ u_w(\theta_w) &\geq c_w(\theta_w') + \check{u}_i\left(\theta_w, \frac{\theta_w'}{\theta_w} l_f(\theta_w')\right) - \check{u}_i(\theta_w', l_f(\theta_w')), \\ \big[u_e(\theta_e) &\geq u_w(\theta_w)\big] \Leftrightarrow \big[i(\theta) = 1\big]. \end{aligned}$$

Second, we can characterize the set of entrepreneurs by a cutoff $e(\theta_w)$ where

$$i(\theta) = \begin{cases} 1 & \text{if} \quad \theta_e \geq e(\theta_w) \\ 0 & \text{if} \quad \theta_e < e(\theta_w). \end{cases} \tag{20}$$

In words, for a given level of labor productivity, all the agents whose entrepreneurial productivity is high enough become workers entrepreneurs and vice versa. We leave a formal proof of this result to Appendix. falta poner anexo

Third, we can replace the set of incentive constraint by their local counterparts which in turn greatly simplifies our analysis. These local incentive constraints in their envelope form are given by

$$u_e(\theta_e) = u(\underline{\theta}) + \int_{\underline{\theta}_e}^{\theta_e} n(s)^\alpha \big[1 - \beta z(s)^\sigma\big] ds, \tag{21}$$

$$u_w(\theta_w) = u(\underline{\theta}) + \int_{\underline{\theta}_w}^{\theta_w} \left(w_i l_i(s) + \chi l(s)^\psi \frac{(l(s) - l_i(s))}{s} - \kappa s^\rho l_i(s)^{1+\rho}\right) ds, \tag{22}$$

together with

$$u_e(e(\theta_w)) = u_w(\theta_w), \forall \theta_w. \tag{23}$$

While the restriction to local incentive constraints are without loss of generality. In the appendix, we provide conditions on fundamentals that will lead to their sufficiency. Furthermore, later in our dynamic model, we use a numerical verification method to verify the validity of this approach.

Fourth, we add the first order conditions for the optimal supply and demand of informal labor:

$$\theta_w(w_i - w_f(1 - T_l'(\theta_w w_f(l - l_i)))) - \kappa \theta_w^{1+\rho} l_i^\rho = 0, \tag{24}$$

$$\left(-w_i + w_f(1 + T_n'((n - n_i - \epsilon) w_f)) - \delta n_i^\gamma\right)\left(1 - T_c'(\pi(\theta_e, n_i, (n - n_i - \epsilon)) - z)\right) = 0 \tag{25}$$

Hence the planner's problem consist on choosing an allocation (1) in order to maximize the concave utilitarian objective function (19) subject to the incentive compatibility conditions (21)-(25) and the feasibility constraints (2).

4.0.1 Optimal control version

Recall an allocation (1) consists of seven functions defined over the type space Θ. The goal here is to reduce the planner's problem size and set it up as an optimal control problem.

Let $f_w(\theta_w)$ and $f_e(\theta_e)$ be the marginal densities and $F_{w|e}$, $F_{e|w}$ the CDF of the marginals. Given an occupational choice $e(\cdot)$, we define the mass of workers with ability θ_w as $h_w(\theta_w)$. Analogously we can define $h_e(\theta_e)$ as the mass of entrepreneurs with skill θ_e. We also define $h(\theta_w)$ as the mass of *agents* that obtain the same utility level as a worker of ability θ_w, it includes all the workers with that skill level plus all the entrepreneurs with skill $\theta_e = e(\theta_w)$. Then it follows that:

$$h_w(\theta_w) = f_w(\theta_w) \cdot F_{e|w}\big(e(\theta_w)\big|\theta_w\big), \tag{26a}$$

$$h_e(\theta_e) = f_e(\theta_e) \cdot F_{w|e}\big(\theta_w|e(\theta_w)\big), \tag{26b}$$

$$h(\theta_w) = h_w(\theta_w) + e'(\theta_w)h_e\big(e(\theta_w)\big). \tag{26c}$$

Since the planner is concave utilitarian,

$$\int_{\underline{\theta_w}}^{\overline{\theta_w}} W\big(u_w(\theta_w)\big)h(\theta_w)d\theta_w, \tag{27}$$

where we used the occupational choice incentive compatibility constraint, equation (23).

Replacing (24) times $\frac{l_i(s)}{s}$ inside (22) we obtain:

$$u_w(\theta_w) = u(\underline{\theta}) + \int_{\underline{\theta}_w}^{\theta_w} \left(\frac{\chi}{s}l(s)^{1+\psi}\right) ds. \tag{28}$$

Also the incentive compatibility constraint in the occupational choice, equation (23), can be written in differential terms as $u_e'\big(e(\theta_w)\big)e'(\theta_w) = u_w'(\theta_w)$, and using the entrepreneur incentive compatibility constraint (21) and the new worker incentive compatibility constraint (28). We obtain:

$$n\big(e(\theta_w)\big)^\alpha\Big[1 - \beta z\big(e(\theta_w)\big)^\sigma\Big]e'(\theta_w) = \left(\frac{\chi}{\theta_w}l(\theta_w)^{1+\psi}\right). \tag{29}$$

The feasibility constraints (2) can be written as isoperimetric condition. Define functions for the aggregate excess supply of goods Y, formal labor L_f and informal labor L_i as follows:

$$Y(\underline{\theta_w}) = L_f(\underline{\theta_w}) = L_i(\underline{\theta_w}) = 0, \tag{30a}$$

their derivatives:

$$\begin{aligned} Y'(\theta_w) = &\left\{e(\theta_w)n\big(e(\theta_w)\big)^\alpha - \delta\frac{n_i\big(e(\theta_w)\big)^{1+\gamma}}{1+\gamma} - \beta\frac{z\big(e(\theta_w)\big)^{1+\sigma}}{1+\sigma} - u_w(\theta_w)\right\} \\ &\cdot e'(\theta_w)h_e(e(\theta_w)) - \left\{u_w(\theta_w) + \chi\frac{l(\theta_w)^{1+\psi}}{1+\psi} + \kappa\frac{\big(\theta_w l_i(\theta_w)\big)^{1+\rho}}{1+\rho}\right\}h_w(\theta_w) \end{aligned} \tag{30b}$$

$$L_f'(\theta_w) = \theta_w l_f(\theta_w)h_w(\theta_w) - n_f\big(e(\theta_w)\big)e'(\theta_w)h_e(e(\theta_w)) \tag{30c}$$

$$L_i'(\theta_w) = \theta_w l_i(\theta_w)h_w(\theta_w) - n_i\big(e(\theta_w)\big)e'(\theta_w)h_e(e(\theta_w)) \tag{30d}$$

and the terminal conditions:

$$Y(\overline{\theta_w}) \geq G \quad L_f(\overline{\theta_w}) \geq 0 \quad L_i(\overline{\theta_w}) \geq 0. \tag{30e}$$

where in equation (30b) we replaced the consumption function from the utility definition as total utility plus disutility of labor in case of being a worker $c(\theta) = u(\theta) + v\big(l(\theta)\big)$ and also the utility of entrepreneurs from the occupational choice incentive compatibility constraint (23).

We define total hours supplied by a worker of type θ_w as $l(\theta_w) = l_f(\theta_w) + l_i(\theta_w)$ and analogously total hours demanded by an entrepreneur of type θ_e as $n(\theta_e) = n_f(\theta_e) + n_i(\theta_e) + \epsilon$. For the planner it is the same to choose functions l, l_i, n, n_i as to choose functions l_f, l_i, n_f, n_i.

Hence the planner's problem is to choose functions $\{u_w, e, l, l_i, n, n_i, z\}$ to maximize equation (27) subject to the labor incentive compatibility constraint(28) and the aggregate isoperimetric constraints (30).

Notice that z is a choice function whose domain is the *entrepreneurial* skill space Θ_e. It only appear in the problem as a composition with the occupational choice indifference function $e(\cdot)$, as in $z\big(e(\cdot)\big)$. Then, we can abuse notation and define it over the *working* skill space Θ_w as $z(\theta_w) = z\big(e(\theta_w)\big)$. Analogously, labor demand functions n and n_i can be defined over Θ_w. This implies every allocation chosen by the planner is equivalent to a set of functions $\{u_w, e, l, l_i, n, n_i, z\}$ all defined over the one-dimensional *worker* skill space Θ_w.

To further reduce the problem, we solve for the profit-hiding function $z(\theta_w)$ in terms of $n(\theta_w)$, $l(\theta_w)$, $e(\theta_w)$ and $e'(\theta_w)$ from equation (29). Also, from equations (24) and (25), we can solve the informal labor supply $l_i(\theta_w)$ and demand $n_i(\theta_w)$ in terms of $z(\theta_w), n(\theta_w), l(\theta_w), e(\theta_w), e'(\theta_w)$ and a constant term w_i which represents the real wage in the informal sector.

Now we are ready to write the optimal control problem. We rename the derivative of the occupation boundary function $p(\theta_w) = e'(\theta_w)$. Dropping the dependence on θ_w to simplify notation, the problem is:

$$\max \int_{\underline{\theta_w}}^{\overline{\theta_w}} \mathbb{1} u_w^{\varphi} h \, d\theta + (1 - \mathbb{1}) u_w(\underline{\theta_w}) h(\underline{\theta_w}), \tag{31a}$$

$$\text{s.t. } u_w' = \frac{\chi}{\theta_w} l^{1+\psi} \tag{31b}$$

$$Y' = \left\{ e n^{\alpha} - \delta \frac{n_i^{1+\gamma}}{1+\gamma} - \beta \frac{z^{1+\sigma}}{1+\sigma} - u_w \right\} p h_e - \left\{ u_w + \chi \frac{l^{1+\psi}}{1+\psi} + \kappa \frac{(\theta_w l_i)^{1+\rho}}{1+\rho} \right\} h_w \tag{31c}$$

$$L_f' = \theta_w (l - l_i) h_w - (n - n_i - \epsilon) p h_e \tag{31d}$$

$$L_i' = \theta_w l_i h_w - n_i p h_e \tag{31e}$$

$$e' = p, \qquad w_i' = 0, \text{ and the boundary conditions} \tag{31f}$$

$$Y(\underline{\theta_w}) = L_f(\underline{\theta_w}) = L_i(\underline{\theta_w}) = 0, \quad Y(\overline{\theta_w}) \geq G, \quad L_f(\overline{\theta_w}) \geq 0, \quad L_i(\overline{\theta_w}) \geq 0. \tag{31g}$$

Falta escribir las boundary conditions.

Where, $\mathbb{1}$ is an indicator variable that describes the planner's objectives. It takes the value of one if the objective function is concave utilitarian and zero if the objective function is Rawlsian.

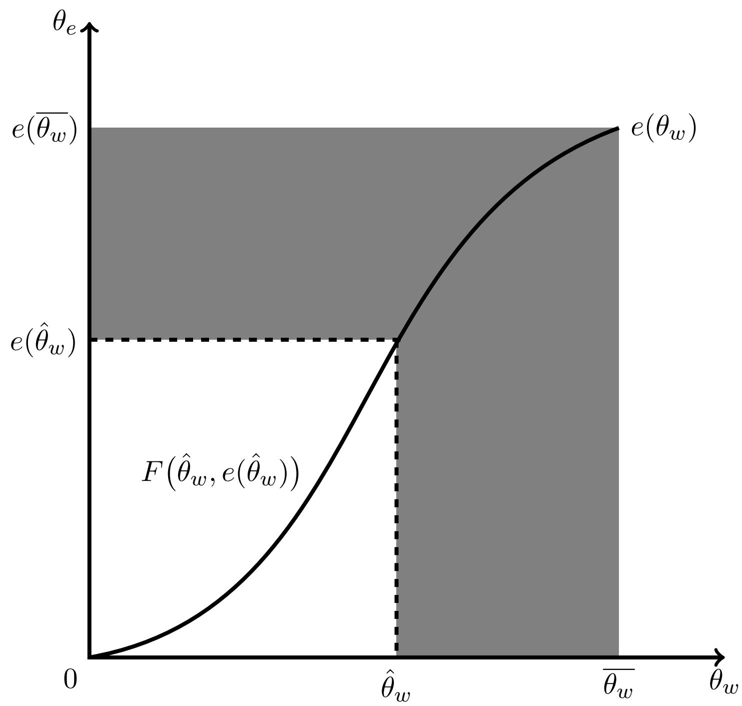
For clarity, we left the profit hiding z, the informal supply l_i and demand n_i in the problem above, but keeping in mind that they are functions of the choice variables.

To set up the Hamiltonian, the state variables are $u_w, Y, L(L_f$ and $L_i), e$ and w_i and the controls are l, n, p. Let $\lambda, \omega_f, \omega_i$ be the multiplier functions associated with the final goods, formal and informal labor constraints. Let μ be the multiplier of the labor IC constraint, ϕ_e the one associated with the differential equality $e'(\theta_w) = p$ and ϕ_w the one associated with the constraint forcing w_i to be constant: $w_i' = 0$. Then the Hamiltonian is:

$$\begin{aligned}\mathcal{H} =& \mathbb{1} u_w^{\varphi} h + \mu \frac{\chi}{\theta_w} l^{1+\psi} + \omega_f[\theta_w l h_w - n p h_e] + (\omega_i - \omega_f)[\theta_w l_i h_w - n_i p h_e] + \phi_e[p] + \phi_w[0] + \\ & \lambda\left\{ e n^{\alpha} - \frac{\delta}{1+\gamma} n_i^{1+\gamma} - \frac{\beta}{1+\sigma} z^{1+\sigma} - u_w \right\} p h_e - \lambda\left\{ u_w + \frac{\chi}{1+\psi} l^{1+\psi} + \frac{\kappa}{1+\rho} (\theta_w l_i)^{1+\rho} \right\} h_w \end{aligned} \tag{32}$$

Figure 7 below shows how the occupational choice is characterized by the increasing function $e(\theta_w)$. The iso-utility curve defined by equation (23) is depicted by the dashed inverted L. Note that, if we take the ocupational choice $e(\theta_w)$ as given, the distribution of workers and entrepreneurs are pinned down and the planner's problem becomes two independent Mirlessian optimal taxation problems. However, the solutions to both workers and entrepreneurs Mirlessian problems lead to an utility schedule for each occupation and define implicitly an occupational choice frontier $e(\theta_w)$ from equation (23). Hence the planner must take into account the agents occupational choice, the extensive margin, in addition to the standard intensive margin found in Mirlessian problems.

Figure 7: Occupational choice



Proposition 1. *In an economy without informality, if the function $T_c(\cdot), T_n(\cdot)$ and $T_l(\cdot)$ implement an equilibrium with taxes maximizing social welfare, they must satisfy the following three formulas*

1.

$$\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)\int_{\theta_w}^{\overline{\theta_w}} h(s)ds = \frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{\epsilon_l}{1+\epsilon_l}\theta_w h_w + \frac{1}{n^\alpha}\frac{\epsilon_z z}{T_c'}T_c'(\cdot)h_e \tag{33}$$

2.

$$\frac{T_n'(\cdot)}{1+T_n'(\cdot)} = \left(1-T_c'(\cdot)\right)\epsilon_z\frac{z}{en^\alpha} \tag{34}$$

3.

$$\underbrace{\frac{\omega}{\lambda}\int_{\underline{\theta_w}}^{s}\mathbb{1}\varphi u_e^{\varphi-1}(1-T_l')l(\theta_w)eh_e(e)d\theta_w}_{\text{Welfare effect}}$$
$$=\underbrace{\int_{\underline{\theta_w}}^{s}\lambda\left[T_l(y_l)-T_c(\pi-z)-T_n(\omega/\lambda n)\right]eg(\theta_w,e)d\theta_w}_{\text{Migration effect}}$$
$$+\underbrace{\lambda\int_{\underline{\theta_e}}^{e(s)}\left(\theta_e n^\alpha-\epsilon_z z\left(1-T_c'(\cdot)\right)-T_n'(\cdot)\omega\epsilon_{n,e}n-T_c'(\cdot)\epsilon_{\pi-z}\left(\pi-z\right)\right)h_e\left(e^{-1}(\theta_e),\theta_e\right)d\theta_e}_{\text{Revenue collection effect}}$$
$$+\underbrace{\lambda(1-T_c'(\cdot))\epsilon_z zeh_e}_{\text{Continuity correction}}.$$

Proof. See appendix A.5. □

Proposition 2. *In an economy with informality, if the function $T_c(\cdot), T_n(\cdot)$ and $T_l(\cdot)$ implement an equilibrium with taxes maximizing social welfare, they must satisfy the following three formulas*

1.

$$\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)\int_{\theta_w}^{\overline{\theta_w}} h(s)ds = \frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{1}{1+\varepsilon_l}\left(\varepsilon_l-\varepsilon_{l_i}\frac{l_i}{l}\right)\theta_w h_w + \frac{\varepsilon_z z}{n^\alpha}h_e. \tag{35}$$

2.

$$\frac{\varepsilon_z z}{n^\alpha} = \frac{eT_n'(\cdot)}{(1+T_n'(\cdot))(1-T_c'(\cdot))}\left(1-\frac{\varepsilon_{n_i}}{\varepsilon_n}\frac{n_i}{n}\right), \tag{36}$$

3.

$$\underbrace{\frac{\omega_f}{\lambda}\int_{\underline{\theta_w}}^{s} \mathbb{1}\varphi u_e^{\varphi-1}(1-T_l'(\cdot))l(\theta_w)h_e(\theta_w,e)\cdot e\,d\theta_w}_{\textit{Welfare effect}} =$$
$$\underbrace{\lambda\int_{\underline{\theta_w}}^{s}[T_l(\cdot)-T_n(\cdot)-T_c(\cdot)]\,g(\theta_w,e(\theta_w))\cdot e\,d\theta_w}_{\textit{Migration effect}}$$
$$+\underbrace{\lambda\int_{\underline{\theta_e}}^{e(s)}\left[\theta_e n^\alpha-(1-T_c'(\cdot))\varepsilon_z z-\omega_f T_n'(\cdot)\varepsilon_{(n-n_i),e}\,(n-n_i)-T_c'(\cdot)\varepsilon_{(\pi-z),e}\,(\pi-z)\right]h_e(e^{-1}(\theta_e),\theta_e)\,d\theta_e}_{\textit{Revenue collection effect}}$$
$$+\underbrace{\lambda(1-T_c'(\cdot))\varepsilon_z z e h_e}_{\textit{Continuity correction}}$$
$$-\underbrace{\frac{\alpha}{\gamma}\int_{\underline{\theta_w}}^{s}\frac{n_i}{n^{1-\alpha}}\left[\lambda T_c'(\cdot)+\omega_f T_n'(\cdot)\frac{1}{\delta n_i^\delta}\right]p\,e\,h_e(\theta_w,e)\,d\theta_w}_{\textit{Informality effect}}. \tag{37}$$

Proof. See appendix A.6. □

Proposition 3. *In an economy with informality, if the function $T_c(\cdot), T_n(\cdot)$ and $T_l(\cdot)$ implement an equilibrium with taxes maximizing social welfare, they must satisfy the following three formulas:*

1.

$$\int_{\theta_w}^{\overline{\theta_w}}\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)h(s)ds=\frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{\varepsilon^{l_f}_{1-T_l'}}{1+\varepsilon^{l_f}_{\theta_w}}\theta_w h_w(\theta_w,e)+\frac{\varepsilon_z z}{n^\alpha}h_e(\theta_w,e) \tag{38}$$

2.

$$\frac{\varepsilon_z z}{n^\alpha}=\frac{e}{(1-T_c'(\cdot))}\cdot\left(\frac{T_n'(\cdot)}{1+T_n'(\cdot)}\right)\left(\frac{-\varepsilon^{n_f}_{1+T_n'}}{\varepsilon^{n_f}_{\theta_e}}\right) \tag{39}$$

3.

$$\lambda\varepsilon^z_{T_c'}\frac{z}{n^\alpha}h_e=\int_{\theta_e}^{\overline{\theta_e}}\lambda\big[T_l(\cdot)-T_c(\cdot)-T_n(\cdot)\big]g(\theta_w,s)\frac{1}{u_e'}ds+\int_{\theta_e}^{\overline{\theta_e}}\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)h_e(\theta_w,s)pds \tag{40}$$

Proof. See appendix A.6. □

5 Calibration

In this section we illustrate the solution method and the calibration strategy for the model. The fundamentals of the model to be calibrated are given by $P=\{\alpha,\beta,\chi,\gamma,\delta,\kappa,\psi,\rho,\sigma,F(\theta_e,\theta_w)\}$.

We assume that the distribution of skills follows a joint log-normal distribution[10]:

$$\begin{bmatrix} \ln(\theta_w) \\ \ln(\theta_e) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_w \\ \mu_e \end{bmatrix}, \begin{bmatrix} \sigma_w^2 & \sigma_{w,e} \\ \sigma_{w,e} & \sigma_e^2 \end{bmatrix}\right) \tag{41}$$

The returns scale parameter is set to $\alpha = 0.8$ as is standard in the literature (Garicano, Lelarge, & Van Reenen, 2016; Lopez & Torres-Coronado, 2018). The remaining parameters $\Phi = \{\beta, \chi, \gamma, \delta, \kappa, \psi, \rho, \sigma, \mu_e, \mu_w, \sigma_w^2, \sigma_e^2, \sigma_{w,e}, \}$ are calibrated to match moments of the data as will be described below.

The model has no closed form solution unless very specific functional forms are imposed to the distribution of skills. For such a reason, we simulate the model for $n = 10,000$ individuals from the assumed distribution of skills and find the corresponding equilibrium wages (w_i, w_f) for a set of parameters.

5.1 Calibration strategy

The calibration strategy consists of choosing parameters to minimize the distance between the simulated moments for $n = 10,000$ individuals and the empirical moments. The following list describes the set of 45 46 moments, composed in six groups:

1. M_1. Proportion of individuals who are workers.
2. M_2^d. Share of income earned by workers for each decile $d = 1..9$ in the income distribution of workers.
3. M_3^d. Share of sales for firms in each decile $d = 1..9$ in the sales distribution of firms.
4. M_4^d. Share of taxes payed by firms in each decile in the sales distribution of firms.
5. M_5^d. Proportion of informal workers for each decile $d = 1...9$ in the income distribution of workers.
6. M_6^d. Proportion of informal workers for each decile $d = 1...9$ in the sales distribution of firms.

The goal of the calibration strategy is to chose parameters to minimize the criterion function

$$J(\Phi) = (T(M;\Phi) - E(M))' \, W \, (T(M;\Phi) - E(M)), \tag{42}$$

where $T(M;\Phi)$ is a 46×1 vector containing the moments predicted by the model, $E(M)$ is a 51×1 vector of empirical moments and W is a 51×51 weight matrix. The weight assigned to the first moment M_1 is 1/6 while the rest of the moments have a weight equal to $\frac{1}{6} \times \frac{1}{9}$ as each one is part of a distribution characterized by 9 moments.

We obtain 1,000 different combinations of the parameter set coming from a Sobol sequence to

[10]See for example (Busso, Neumeyer, & Spector, 2012).

obtain a well-balanced coverage of the parameter set. The calibrated parameters are included in Table 9.

Table 9: Calibration results

Parameter	Estimate
β	0.2135
χ	2.0192
γ	0.7341
δ	0.12873
κ	0.1021
ψ	0.4528
ρ	0.0912
σ	0.1827
μ_e	1.2528
μ_w	1.7626
σ_w^2	1.0921
σ_e^2	1.1675
$\sigma_{w,e}$	0.2782

5.2 Model Fit

In this subsection we present the model fit comparing some of the empirical moments with their theoretical counterparts. We show that for the case of production levels, tax payments, income distribution, and informal labor supply, the model does a good job fitting aggregate moments as well as their relationship with firms' production or households' income levels.

Figure 8: Distribution of production by deciles

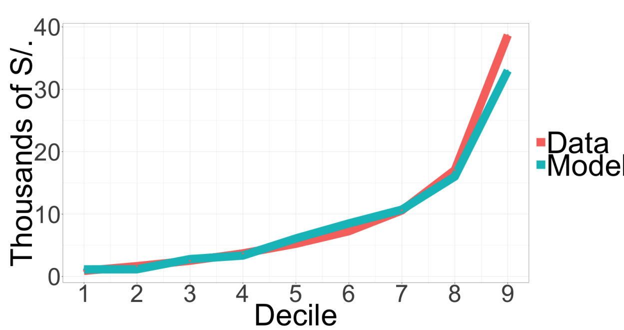


Figure 9: Income distribution. Households' earnings by income decile

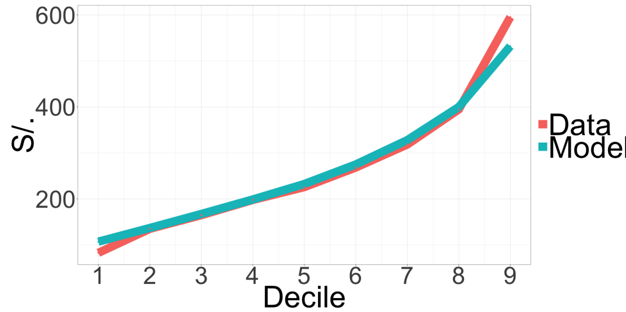


Figure 10: Tax payments. Proportion of total taxes payed by firms in each production decile.

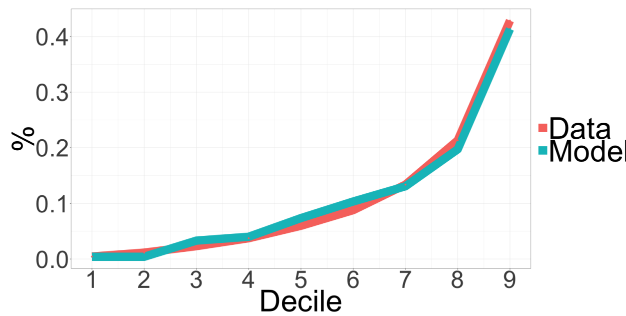
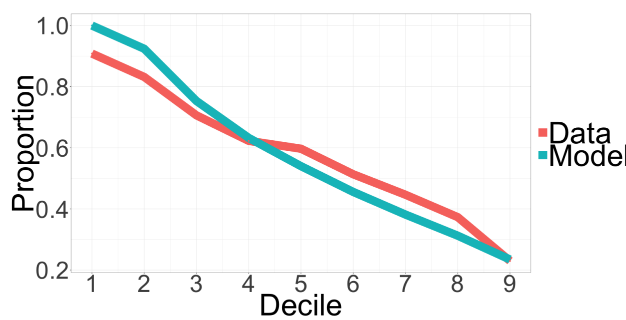


Figure 11: Informal labor supply



6 Conclusion

In this paper, we develop a theory of optimal taxation in an economy with an informal sector. An important challenge in any empirical work regarding informality is the data, since by definition informal activities are not observed by the authorities. We overcome this limitation by combining multiple sources of information including the Economic Census of Peru, administrative tax records from tax authorities, and a nationally representative household survey, to obtain a unique characterization of the informal economy in Peru.

We incorporate the main empirical features that we observe in the data, in a general equilibrium model with informality. Informal workers are heterogeneous in terms of occupation: about 30% are employees, another 30% are self-employed, 4% are employers and the rest are not remunerated for their work. We build a model that accounts for this heterogeneity, and crucially, allows for different skills when an agent chooses to be self-employed or employer as opposed to work as an employee.

In our model, entrepreneurs hire formal and informal workers and do not pay payroll taxes on their informal workforce. On the other hand, workers do not pay taxes on their income generated in the informal economy. In line with the data, we explicitly acknowledge the fact that entrepreneurs can under-report their production to avoid corporate taxes. The data allows us to separately identify misreporting from behavioral responses to corporate taxes, and hence to measure the effect of such taxes on welfare with higher precision relative to previous work. Our model is able to replicate the main features of the data closely.

As occupational choice results in potentially different levels of skill for the same individual, the mechanism design problem cannot be solved using standard tools. We develop a method to simplify the problem and write it as an optimal control problem. This permits to deduce simple tax formulas from the optimally conditions of the problem.

We provide optimal tax formulas, that depend crucially on the elasticity of informal labor to taxes. We show that, when individuals are free to chose their occupational sector, the tax functions for entrepreneurs and for workers are jointly determined, and solving the problems independently yields to misleading solutions.

In the next steps we need to calculate the welfare effects of having a tax policy that differs from the optimal tax scheme. This is the next step to apply this methodological proposal.

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A Appendix

A.1 Model solution with constant marginal tax rates

The solution to the entrepreneur problem when marginal tax rates are constant is given by equations 8 - 10:

$$n_i + n_f + \epsilon = \left(\frac{\alpha \theta_e}{w_f (1 + T_n')} \right)^{\frac{1}{1-\alpha}},$$

$$n_i = \left(\frac{w_i - w_f (1 + T_n')}{\delta} \right)^{\frac{1}{\gamma}},$$

$$z = \left(\frac{T_c'}{\beta} \right)^{\frac{1}{\sigma}}.$$

And thus, plugging these equations into 3 we obtain an expression for $\Pi(\theta_e; w_f, w_i)$. Falta explicar qué es Π, no está definido antes.

The solution to the problem of the worker is given by:

$$l_i = \min \left\{ \left(\frac{\theta_w w_i}{\kappa} \right)^{\frac{1}{\rho}}, 1 \right\}, \tag{43}$$

$$l_f = 1 - \min \left\{ \left(\frac{\theta_w w_i}{\kappa} \right)^{\frac{1}{\rho}}, 1 \right\}. \tag{44}$$

con las ecuaciones que tenemos, se obtiene otro valor para l_i, se tendría:

$$l_i = \min \left\{ \left(\frac{w_i - w_f (1 - T_l')}{\kappa \theta_w^{\rho}} \right)^{\frac{1}{\rho}}, 1 \right\}, \tag{45}$$

$$l_f = 1 - \min \left\{ \left(\frac{w_i - w_f (1 - T_l')}{\kappa \theta_w^{\rho}} \right)^{\frac{1}{\rho}}, 1 \right\}. \tag{46}$$

And the value of the problem is given by plugging these equations into 11 to obtain $V(\theta_w; w_f, w_i)$. Falta explicar qué es V, no está definido antes.

Entrepreneurial decision is given by

$$i(\theta_e, \theta_w; w_i, w_f) = \mathbb{1}\{\Pi(\theta_e; w_f, w_i) > V(\theta_w; w_f, w_i)\}. \tag{47}$$

Wages are found by the market clearing conditions

$$\int_{\Theta} n_i(\theta_e) i(\theta_e, \theta_w; w_f, w_i) dF(\Theta) = \int_{\Theta} \theta_w l_i(\theta_w, w_i)(1 - i(\theta_e, \theta_w; w_i, w_f)) dF(\Theta), \tag{48}$$

$$\int_{\Theta} n_f(\theta_e) i(\theta_e, \theta_w; w_f, w_i) dF(\Theta) = \int_{\Theta} \theta_w l_f(\theta_w, w_i)(1 - i(\theta_e, \theta_w; w_i, w_f)) dF(\Theta). \tag{49}$$

A.2 Taxes in Peru

In this section we describe how payroll, corporate and personal income tax operate in the Peruvian economy.

A.2.1 Payroll taxes

Employers pay additional charges for each worker hired in the form of holidays, contributions to unemployment insurance (CTS), bonus, contribution to health insurance, and family subsidy. Employers are not required to make contributions to employees pension plans. The amount payed by the employer varies according to the size of the firm, as is specified in Table 10.

Table 10: Payroll extra charges payed by employer (yearly)

	Holidays	CTS	Bonus	Health insurance	Family subsidy
General	4 weeks	1 monthly wage	2 monthly wages	9%	10% of minimum wage
Medium	2 weeks	0.5 monthly wage	1 monthly wage	9%	0
Micro	2 weeks	0	0	180S/.	0

Note: CTS stands for "Compensación por Tiempo de Servicio". This is a contribution made by the employer to an unemployment insurance account accessible to the employee whenever the employment relationship ends. Family subsidies are given to workers with at least one child under 18 or under 24 who is studying. 86% of the sample analyzed is eligible for the family subsidy.

We compute the total monthly cost of hiring a worker w_T depending on firm's size in terms of the gross monthly wage w_G. We assume that 20 days is equivalent to 2/3 of a month and a month consists of 4.3 weeks and we consider the minimum monthly wage in Peru which was 530S/.

$$w_T^{micro} = \underbrace{w_G}_{\text{Gross wage}} + \underbrace{\left(\frac{1}{12} \times \frac{1}{2.15}\right) \times w_G}_{\text{holidays}} + \underbrace{15}_{\text{Health insurance}} = 1.038 \times w_G + 15, \tag{50}$$

$$\begin{aligned} w_T^{medium} = &\underbrace{w_G}_{\text{Gross wage}} + \underbrace{\left(\frac{1}{12} \times \frac{1}{2.15}\right) \times w_G}_{\text{holidays}} + \underbrace{w_G \times \frac{1}{24}}_{\text{CTS}} + \underbrace{w_G \times \frac{1}{12}}_{\text{bonus}} \\ &+ \underbrace{w_G \times 0.09}_{\text{Health Insurance}} = 1.25 \times w_G, \end{aligned} \tag{51}$$

$$\begin{aligned} w_T^{General} = &\underbrace{w_G}_{\text{Total wage}} + \underbrace{w_G \times \left(\frac{1}{12}\right) \times \left(\frac{4}{4.3}\right)}_{\text{holidays}} + \underbrace{w_{week} \times \frac{1}{12}}_{\text{CTS}} + \underbrace{w_G \times \frac{2}{12}}_{\text{bonus}} \\ &+ \underbrace{w_G \times 0.09}_{\text{Health Insurance}} + \underbrace{0.1 \times 530}_{\text{Family Subsidy}} = 1.42 \times w_G + 53. \end{aligned} \tag{52}$$

Micro firms have yearly sales under 517,500S/. and Medium firms 5,865,000S/. We define the PT as the total extra. Defining PT as the payroll taxes for a worker earning the average wage of 785/S:

$$\text{Labor cost} = \begin{cases} 1.038 + \frac{15}{785} - 1 = 5.71\%, & \text{if sales}<517{,}500 \\ 25\%, \quad \text{if} \quad 517,500 \leq & \text{sales} \leq 5,865,000 \\ 1.42 + \frac{53}{785} - 1 = 42\% & \text{otherwise} \end{cases} \tag{53}$$

We report the distribution of firms in each of the payroll tax regimes in Table 11. Only 8% of firms lie in the top two payroll tax regimes with only 1% corresponding to the top one.

Table 11: Firms: revenues and payroll tax regimes

Revenue	Proportion
(0,517,500]	0.92
(517,500,5,865,000]	0.07
5,865,000+	0.01

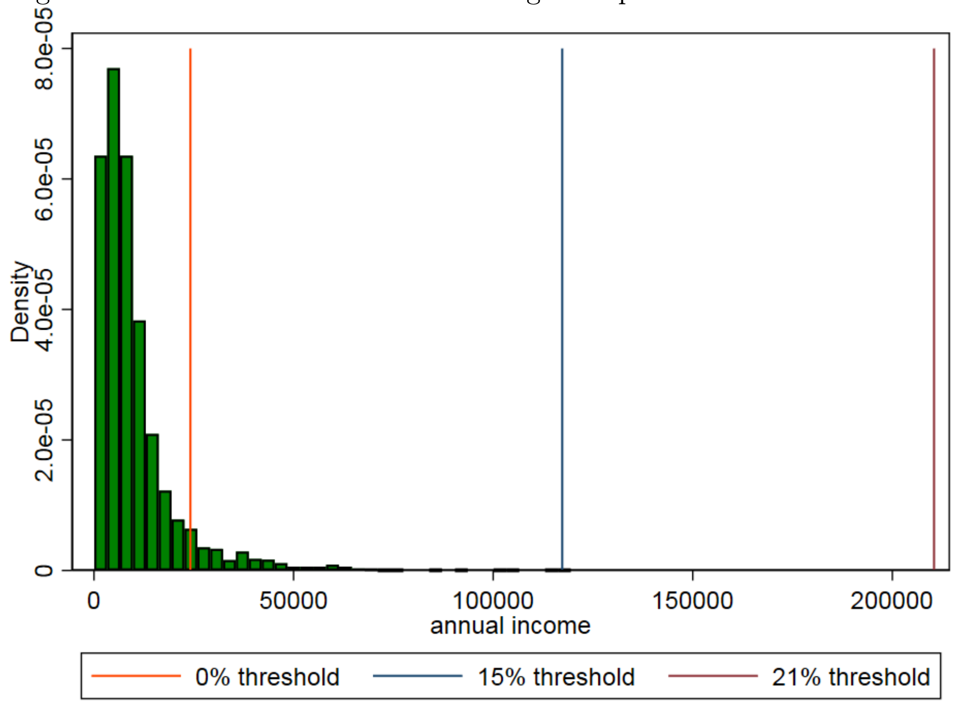
A.2.2 Personal Income Taxes and Government Transfers

There are five different regimes for the personal income tax according to the total annual income perceived. The schedule is given by the following function.

$$\text{Tax rate} = \begin{cases} 0\% \text{ if annual income} \leq 24,150S/. \\ 15\% \text{ if } 24,150S/. < \text{annual income} \leq 117,300S/. \\ 21\% \text{ if } 117,300S/. < \text{annual income} \leq 210,450/. \\ 30\% \text{ if annual income} \geq 210,450S/. \end{cases} \tag{54}$$

We report the distribution of annual earnings, together with the different thresholds for income eligibility in Figure 12.Approximately 92% of individuals do not have to pay personal income tax, 7.4% pay 15%, 0.02% pay 21% and there are no individuals in the sample who are located within the 30% tax rate.

Figure 12: Distribution of Annual earnings and personal income tax rates



Note: The vertical lines represent the maximum income to be eligible for a given tax rate.

The system of government transfers to households is composed by five programs: retirement pension, disability pensions, pensions to widows, pensions to orphans, pension to other descendants of the pensioner, and "Red Juntos". "Red Juntos" is a conditional cash transfer program for poor families in Peru. In 2007, 638 districts were included as beneficiaries of this program. Households from Lima were not eligible for this program.

In addition to these transfers, the government provides health insurance for a fraction for the poorest population through the SIS (Integrated Health Insurance)[11]. Individuals who are not eligible for free health insurance through the SIS can pay a monthly fee of 30S/. In ENAHO we have information for those who have access to the SIS. We assume that this is equivalent to a monthly transfer or 30S/.

Firms who employ more than twenty people are required to distribute between 5% and 10% among their employees. Information about the disbursement of these share of the profits is also available in the ENAHO survey. In principle we can consider these payments as part of the transfer schemes from the government as firms face a higher corporate income tax rate that ends up being transfered to the workers.

[11]"Seguro Integrado de Salud" in Spanish.

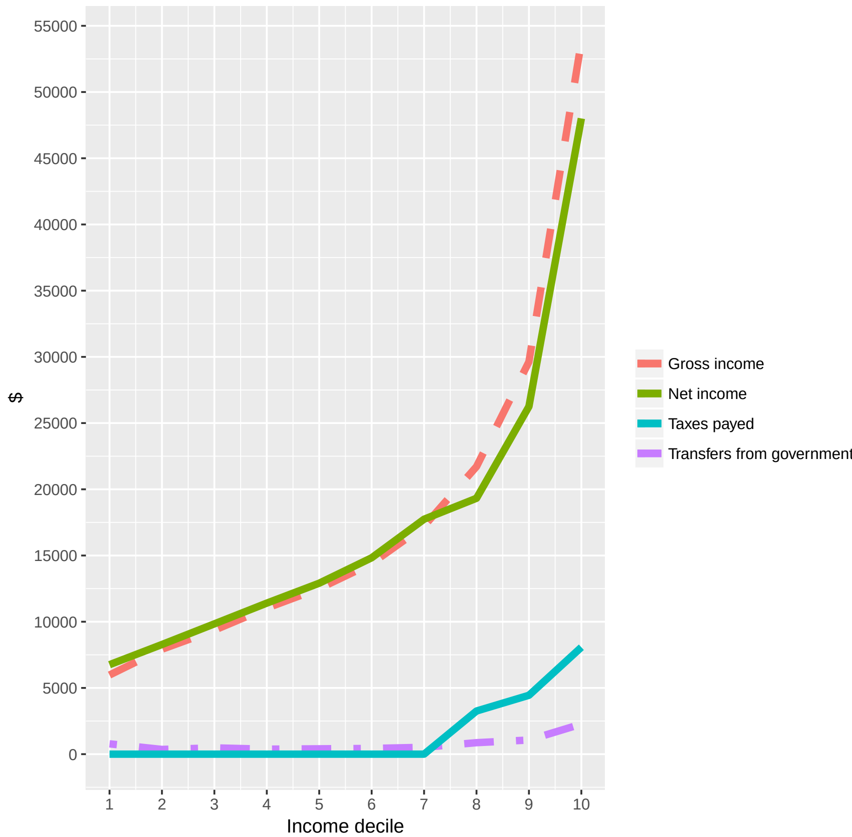
Table 12: Distribution of average monthly transfers per capita to households, and SIS membership, by labor income per capita

Decile	Labor Income	Direct Transfers	Households in SIS	Income from profit sharing
1.00	63.36	56.07	0.30	0.00
2.00	137.84	21.99	0.23	0.88
3.00	186.56	30.68	0.24	1.55
4.00	238.16	24.01	0.16	2.44
5.00	289.53	22.32	0.18	5.94
6.00	350.13	26.06	0.12	6.29
7.00	438.01	26.36	0.11	13.42
8.00	562.08	46.81	0.07	23.22
9.00	796.91	47.92	0.04	39.75
10.00	2034.87	55.72	0.02	132.84

Note: Households are considered members of the SIS if at least one person from the household benefits from this service.

Direct transfers do not seem to be entirely progressive. The pension system benefits both, poor and rich households almost equally and the profit sharing rule benefits only the richest households. Out of the three transfers systems, only access to health insurance seems to be targeted to poor households. We plot the distribution of earnings for formal workers after transfers and taxes in Figure 13.

Figure 13: Income distribution after transfers and taxes



A.3 Corporate tax regimes in Peru

In 2007, firms payed corporate income taxes according to one of the three tax regimes existing at the moment. These regimes are RUS[12], RER[13], and the general regime for corporate income tax. RUS is designed for natural persons with entrepreneurial activities. Eligibility into this regime requires having monthly income under 30,000 soles, value of assets of less than 70,000 soles, and having all operations of the business in one location at most, among others[14]. Under this regime, the corporate income tax and the value added tax is replaced by a monthly quota determined by the monthly income of the business as described in Table 13. Moreover, businesses are exempt from paying value added taxes which had a rate of 19% in 2007, and are not required to have updated financial ledgers[15].

[12]"Régimen único simplificado" in Spanish, which translates to "unique simplified regime".

[13]"Régimen Especial de Impuesto de renta" which translates to "Special corporate income tax rate".

[14]Activities such as transportation, gambling, finance, travel agencies, real estate, or commercialization of oil and hydrocarbon are excluded from the RUS. Businesses who export part of their merchandise are also not eligible to pay taxes according to this regime.

[15]In addition to the categories mentioned in Table 13 there is a special category called "Nuevo RUS" (New RUS) directed to agricultural businesses with annual income under 60,000 S/. As we do not consider the agricultural sector

Table 13: RUS tax scheme

Category	Monthly income (Soles)	Monthly payments (Soles)
1	5,000	20
2	8,000	500
3	13,000	200
4	20,000	400
5	30,000	600

As the RUS, the RER is a tax regime designed to small businesses. However, RER is targeted exclusively to legal entities as well as legal persons. To be eligible in the RER, a business should have net annual income of no more than 360,000 S/.[16], the total value of the assets should be under 87,500 S/. and the total amount of annual purchases, excluding acquisition of fixed assets, should also be under 360,000 S/. As in the case of the RUS, the RER also excludes some economic activities[17].

Businesses registered under the RER scheme are required to pay value added tax, and to keep updated financial ledgers. Under this regime, the tax on profits is substituted by a tax on net income of the business. Businesses operating in the service sector pay 2.5% of their monthly net income and the corresponding rate for businesses operating in commerce or industry is of 1.5%.

All businesses not eligible for either the RUS or the RER scheme, are subject to the regulation of the general regime of taxation. Businesses registered in the general regime are required to pay 30% of their profits at the end of the year. We summarize the tax regimes with its corresponding obligations and requirements in Table **??**. No está esta tabla

Monthly tax obligation for businesses

$$\text{Taxes} = \begin{cases} 20S/.\text{ if total income} \leq 5,000S/. \\ 50S/.\text{ if total income} \leq 8,000S/. \\ 200S/.\text{ if total income} \leq 13,000S/. \\ 400S/.\text{ if total income} \leq 20,000S/. \\ 600S/.\text{ if total income} \leq 30,000S/. \\ 1.5\% - 2.5\%\text{ of total income, depending on sector, if total income} \leq 30,000S/. \\ 30\%\text{ of profits if total income} > S/.30,000 \\ 35\% - 40\%\text{ of profits if total income} > S/.30,000\text{ and more than 20 workers} \end{cases} \quad (55)$$

in the analysis we do not go much into the detail of this category.

[16]Net annual income is equal to gross annual income less discounts, returns, or other similar practices done by businesses.

[17]Construction, transportation, finance, gambling, travel agencies, real estate, judiciary services, accounting, architecture, and business consulting are all excluded from the RER. Doctors, dentists, and veterinarians are also ineligible.

In Table 14 we report the proportion of firms in each corporate profit tax regime. Firms with monthly revenues under 30,000 S/. pay a monthly fee that depends on the revenue. As we only consider corporate profit taxes in the model, we estimate the equivalent profit tax rate as the rate that would generate the same taxes on corporate profits as the monthly fee.

Table 14: Firms: revenue, profits and corporate profit tax equivalent

Revenue	Workers	Proportion of firms	Average profits	Equivalent profit tax rate
(0,5,000]		0.7	594.66	0.03
(5,000,8,000]		0.09	1621.94	0.03
(8,000,13,000]		0.06	2384.7	0.08
(13,000,20,000]		0.03	3328.14	0.12
(20,000,30,000]		0.02	4772.87	0.13
30,000+	<20	0.08	13757.55	0.3
30,000+	>=20	0.02	60103.27	0.38

A.4 Solution to the planner's optimal control problem

As stated in equation 32, the Hamiltonian associated to the Utilitarian planner's problem can be written as,

Control variables: l, n, p

State variables: $u_w[\mu], Y[\lambda], L[\omega_f], L_i[\omega_i], e[\phi_e], w_i[\phi_w]$

$$\begin{aligned}\mathcal{H} =& \mathbb{1}u_w^{\varphi}h + \mu\frac{\chi}{\theta_w}l^{1+\psi} + \omega_f[\theta_w l h_w - nph_e] + (\omega_i - \omega_f)[\theta_w l_i h_w - n_i ph_e] + \phi_e[p] + \phi_w[0] + \\ &\lambda\left\{en^{\alpha} - \frac{\delta}{1+\gamma}n_i^{1+\gamma} - \frac{\beta}{1+\sigma}z^{1+\sigma} - u_w\right\}ph_e - \lambda\left\{u_w + \frac{\chi}{1+\psi}l^{1+\psi} + \frac{\kappa}{1+\rho}(\theta_w l_i)^{1+\rho}\right\}h_w\end{aligned}$$

where the following shorthands where used,

$$z = \left(\frac{1}{\beta} - \frac{\chi l^{1+\psi}}{\beta\theta_w p n^{\alpha}}\right)^{\frac{1}{\sigma}} \tag{56}$$

$$l_i = \left(\frac{\theta_w w_i - \chi l^{\psi}}{\kappa\theta_w^{1+\rho}}\right)^{\frac{1}{\rho}} \tag{57}$$

$$n_i = \left(\frac{\left(\alpha e n^{\alpha-1} - w_i\right)(1-\beta z^{\sigma})}{\delta}\right)^{\frac{1}{\gamma}} = \left(\frac{\left(\alpha e n^{\alpha-1} - w_i\right)}{\delta}\frac{\chi l^{1+\psi}}{\theta_w p n^{\alpha}}\right)^{\frac{1}{\gamma}} \tag{58}$$

The corresponding optimality conditions are as follow,

$$
\begin{aligned}
\{l\}: 0 =& \mu(1+\psi)\frac{\chi}{\theta_w}l^{\psi} + w_f\theta_w h_w + (\omega_i - \omega_f)\Big[\theta_w\frac{\partial l_i}{\partial l}h_w - \frac{\partial n_i}{\partial l}ph_e\Big] \\
&+ \lambda\Big[-\delta n_i^{\gamma}\frac{\partial n_i}{\partial l} - \beta z^{\sigma}\frac{\partial z}{\partial l}\Big]ph_e - \lambda\Big[\chi l^{\psi} + \kappa\theta_w^{1+\rho}l_i^{\rho}\frac{\partial l_i}{\partial l}\Big]h_w \\
\{n\}: 0 =& -\omega_f ph_e - (\omega_i - \omega_f)\frac{\partial n_i}{\partial n}ph_e + \lambda\Big[\alpha e n^{\alpha-1} - \delta n_i^{\gamma}\frac{\partial n_i}{\partial n} - \beta z^{\sigma}\frac{\partial z}{\partial n}\Big]ph_e \\
\{p\}: 0 =& \mathbb{1}u_w^{\varphi}h_e - \omega_f n h_e + (\omega_i - \omega_f)\Big[-n_i h_e - ph_e\frac{\partial n_i}{\partial p}\Big] + \phi_e - \lambda\Big[\delta n_i^{\gamma}\frac{\partial n_i}{\partial p} + \beta z^{\sigma}\frac{\partial z}{\partial p}\Big]ph_e \\
&+ \lambda\Big[en^{\alpha} - \frac{\delta}{1+\gamma}n_i^{1+\gamma} - \frac{\beta}{1+\sigma}z^{1+\sigma} - u_w\Big]h_e \\
\{e\}: -\phi_e' =& \frac{\partial h_e}{\partial e}\left((\mathbb{1}u_w^{\varphi} - \lambda u_w) - \omega_f n - (\omega_i - \omega_f)n_i + \lambda\left[en^{\alpha} - \frac{\delta}{1+\gamma}n_i^{1+\gamma} - \frac{\beta}{1+\sigma}z^{1+\sigma}\right]\right)p \\
&+ \frac{\partial h_w}{\partial e}\left((\mathbb{1}u_w^{\varphi} - \lambda u_w) + \omega_f l\theta_w + (\omega_i - \omega_f)l_i\theta_w - \lambda\left[\frac{\chi}{1+\psi}l^{1+\psi} + \frac{\kappa}{1+\rho}(\theta_w l_i)^{1+\rho}\right]\right) \\
&+ \lambda n^{\alpha}ph_e - \frac{\partial n_i}{\partial e}\left[\lambda\delta n_i^{\gamma} + (\omega_i - \omega_f)\right]ph_e \\
\{u_w\}: -\mu' =& \mathbb{1}(\varphi u_w^{\varphi-1}h) - \lambda ph_e - \lambda h_w = h(\mathbb{1}\varphi u_w^{\varphi-1} - \lambda) \\
\{w_i\}: -\phi_w' =& (\omega_i - \omega_f)\left[\theta_w h_w\frac{\partial l_i}{\partial w_i} - ph_e\frac{\partial n_i}{\partial w_i}\right] - \lambda\left[\delta n_i^{\gamma}\frac{\partial n_i}{\partial w_i}ph_e + \kappa\theta_w^{1+\rho}l_i^{\rho}\frac{\partial l_i}{\partial w_i}h_w\right] \\
\{Y\}: -\lambda' =& 0 \\
\{L_i\}: -\omega_i' =& 0 \\
\{L_f\}: -\omega_f' =& 0
\end{aligned}
$$

And the transversality conditions.

A.5 Derivation of the Optimal Tax Formulas Without Informality

Without informality, the planner's problem is as follows:

$$
\max \int_{\underline{\theta_w}}^{\overline{\theta_w}} \mathbb{1}u_w^{\varphi}h\,d\theta + (1-\mathbb{1})u_w(\underline{\theta_w})h(\underline{\theta_w}), \tag{59a}
$$

$$
\text{s.t. } u_w' = \frac{\chi l^{1+\psi}}{\theta_w} \tag{59b}
$$

$$
Y' = en^{\alpha}ph_e - \frac{\beta z^{1+\sigma}}{1+\sigma}ph_e - u_w h - \frac{\chi l^{1+\psi}}{1+\psi}h_w \tag{59c}
$$

$$
L' = \theta_w l h_w - nph_e(e) \tag{59d}
$$

$$
e' = p, \text{ and the boundary conditions} \tag{59e}
$$

$$
Y(\underline{\theta_w}) = L(\underline{\theta_w}) = 0, e(\underline{\theta_w}) = \underline{\theta_e} \quad Y(\overline{\theta_w}) \geq G, \quad L(\overline{\theta_w}) \geq 0 \quad e(\overline{\theta_w}) = \overline{\theta_e}. \tag{59f}
$$

Where $z(l, n, p; \theta_w)$ is implicitly defined by,

$$
\frac{\chi l^{1+\psi}}{\theta_w} = pn^{\alpha}(1 - \beta z^{\sigma}). \tag{60}
$$

From equation (60) it follows that,

$$\frac{\partial z}{\partial l} = -\frac{\chi(1+\psi)l^{\psi}}{\theta_w p n^{\alpha}\beta\sigma z^{\sigma-1}} \tag{61a}$$

$$\frac{\partial z}{\partial n} = \frac{\alpha(1-\beta z^{\sigma})}{n^{\alpha}\beta\sigma z^{\sigma-1}} \tag{61b}$$

$$\frac{\partial z}{\partial p} = \frac{1-\beta z^{\sigma}}{p\sigma\beta z^{\sigma-1}} \tag{61c}$$

The state variables are u_w, Y, L, e and the controls are l, n, p. Let $\mu, \lambda, \omega, \phi_e$ be the multiplier functions associated with u_w, Y, L, e, respectively. The Hamiltonian is:

$$\begin{aligned}\mathcal{H} =& \mathbb{1}u_w^{\varphi}h + \mu\frac{\chi l^{1+\psi}}{\theta_w} + \lambda\left[en^{\alpha}ph_e - \frac{\beta z^{1+\sigma}}{1+\sigma}ph_e - u_w h - \frac{\chi l^{1+\psi}}{1+\psi}h_w\right] \\ &+ \omega\big[\theta_w l h_w - nph_e(e)\big] + \phi_e[p]\end{aligned} \tag{62}$$

We define the value for the planner of a worker, adjusted by the surplus of goods it gives the planner, and analogously for an entrepreneur:

$$V_w(\theta_w) = \mathbb{1}u_w^{\varphi} + \left[\theta_w l\omega - \lambda\left(u_w + \frac{\chi l^{1+\psi}}{1+\psi}\right)\right] = \mathbb{1}u_w^{\varphi} + \lambda T_l(\cdot), \tag{63a}$$

$$\begin{aligned}\hat{V}_e(\theta_e) &= \mathbb{1}u_e(\theta_e)^{\varphi} + \lambda\theta_e n_e(\theta_e)^{\alpha} - \lambda\frac{\beta z_e(\theta_e)^{1+\sigma}}{1+\sigma} - \omega n_e(\theta_e) - \lambda u_e(\theta_e) \\ &= \mathbb{1}u_e(\theta_e)^{\varphi} + \lambda\left(T_n(\cdot) + T_c(\cdot)\right)\end{aligned} \tag{63b}$$

and in terms of θ_w set $V_e(\theta_w) = \hat{V}_e\big(e(\theta_w)\big)$ as:

$$V_e(\theta_w) = \mathbb{1}u_w^{\varphi} + \lambda e n^{\alpha} - \lambda\frac{\beta z^{1+\sigma}}{1+\sigma} - \omega n - \lambda u_w \tag{63c}$$

The Hamiltonan $\mathcal{H}$ can be rewritten using the planner valuations of the agents utility and output (equations 63) as:

$$\mathcal{H} = V_w h_w + V_e p h_e + \mu\frac{\chi l^{1+\psi}}{\theta_w} + \phi_e[p] \tag{64}$$

Notice that neither of Y, L appears on the Hamiltonian $\mathcal{H}$, hence the state optimality conditions yield:

$$\lambda' = \omega' = 0. \tag{65a}$$

which imply those multipliers functions are constant. The optimality conditions for the workers utility profile u_w is:

$$\frac{\partial\mathcal{H}}{\partial u_w} = -\mu' = h(\mathbb{1}\varphi u_w^{\varphi-1} - \lambda). \tag{65b}$$

Now, notice that the defined distribution functions $h_w(\theta_w), h_e(\theta_w)$ all depend on e but only through the value of $e(\theta_w)$ and not $e'(\theta_w)$. Hence, using the short version of the Hamiltonian given by equation (64)

$$\frac{\partial \mathcal{H}}{\partial e} = -\phi_e' = V_w \frac{\partial h_w(e)}{\partial e} + \frac{\partial V_e}{\partial e} p h_e(e) + V_e p \frac{\partial h_e(e)}{\partial e} \tag{65c}$$

The optimality condition with respect to labor allocation to firms n is:

$$\frac{\partial \mathcal{H}}{\partial n} = 0 = \lambda \left[e\alpha n^{\alpha-1} - \frac{\omega}{\lambda} - \beta z^{\sigma} \frac{\partial z}{\partial n} \right] p h_e(e). \tag{65d}$$

With respect to the labor supply $\frac{\partial \mathcal{H}}{\partial l}$:

$$0 = \frac{\mu}{\theta_w} \chi(1+\psi) l^{\psi} - \left[\lambda \chi l^{\psi} - \omega \theta_w\right] h_w - \lambda \beta z^{\sigma} \frac{\partial z}{\partial l} p h_e(e). \tag{65e}$$

Last, the optimality condition for the derivative of the choice function $\frac{\partial \mathcal{H}}{\partial p}$ is:

$$0 = \frac{\partial V_e}{\partial p} p h_e(e) + V_e h_e(e) + \phi_e \tag{65f}$$

A.5.1 Labor supply optimality condition

From the implementation of the worker's problem we have

$$\frac{\theta_w \omega}{\lambda} - \chi l^{\psi} = T_l'(\cdot) \frac{\theta_w \omega}{\lambda},$$

hence $\frac{\lambda \chi l^{\psi}}{\theta_w \omega} = 1 - T_l'(\cdot)$. Also $\frac{1}{\epsilon_l} = \psi$, where $\epsilon_l = \frac{\partial l}{\partial(1-T_l'(\cdot))} \frac{(1-T_l'(\cdot))}{l}$ is the price elasticity of labor. Hence we have:

$$\left[1 - T_l'(\cdot)\right] \left[1 + \frac{1}{\epsilon_l}\right] = \frac{\lambda \chi l^{\psi}}{\theta_w \omega}(1+\psi) \tag{66}$$

Since μ is the multiplier function on the labor IC constraint, we have $\mu(\overline{\theta_w}) = 0$ and hence from the optimality condition for u_w, equation (65b) we get

$$\mu(\theta_w) = \int_{\theta_w}^{\overline{\theta_w}} -\frac{d\mu(s)}{ds} ds = \int_{\theta_w}^{\overline{\theta_w}} (\mathbb{1}\varphi u_w^{\varphi-1} - \lambda) h(s) ds \tag{67}$$

Divide the optimality condition for labor supply equation (65e) by ω and replace from equation (66) and divide by $\left[1 - T_l'(\cdot)\right]\left[1 + \frac{1}{\epsilon_l}\right]$, then use equation (67) to obtain:

$$\begin{aligned} -\frac{\mu}{\lambda} &= \frac{T_l'(\cdot)}{1-T_l'(\cdot)} \frac{\epsilon_l}{1+\epsilon_l} \theta_w h_w - \frac{\lambda}{\omega} \frac{1}{1-T_l'(\cdot)} \frac{\epsilon_l}{1+\epsilon_l} \beta z^{\sigma} \frac{\partial z}{\partial l} p h_e \\ \int_{\theta_w}^{\overline{\theta_w}} \left(1 - \mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right) h(s) ds &= \frac{T_l'(\cdot)}{1-T_l'(\cdot)} \frac{\epsilon_l}{1+\epsilon_l} \theta_w h_w - \frac{\lambda}{\omega} \frac{1}{1-T_l'(\cdot)} \frac{\epsilon_l}{1+\epsilon_l} \beta z^{\sigma} \frac{\partial z}{\partial l} p h_e \end{aligned} \tag{68}$$

From the implementation of the entrepreneur's problem (at $\theta_e = e(\theta_w)$) we have:

$$\left[e\alpha n^{\alpha-1} - \frac{\omega}{\lambda}\left(1 + T_n'(\cdot)\right)\right]\left(1 - T_c'(\cdot)\right) = 0 \tag{69a}$$

As long as $T_c'(\cdot) \neq 1$,

$$\alpha e n^{\alpha-1} - \frac{\omega}{\lambda} = T_n'(\cdot)\frac{\omega}{\lambda} \tag{69b}$$

And,

$$T_c'(\cdot) = \beta z^{\sigma} \tag{69c}$$

where $\pi(\cdot) = en^{\alpha} - \frac{\omega}{\lambda}n - T_n(\cdot)$

Deriving (60) we get,

$$\frac{(1+\psi)\chi l^{\psi}}{\theta_w} = -pn^{\alpha}\beta\sigma z^{\sigma-1}\frac{\partial z}{\partial l} \tag{70}$$

Combining the equation (70) above with (66) and (69c) we can write,

$$\frac{\partial z}{\partial l} = -\frac{\frac{\omega}{\lambda}(1-T_l'(\cdot))(1+1/\epsilon_l)}{pn^{\alpha}\beta\sigma z^{\sigma-1}} \tag{71}$$

From (60),

$$pn^{\alpha} = \frac{\chi l^{1+\psi}}{\theta_w(1-\beta z^{\sigma})} \tag{72}$$

Using again the implementation conditions (66) and (69c), the equation above can be written as,

$$p = \frac{(1-T_l'(\cdot))l}{(1-T_c'(\cdot))n^{\alpha}}\frac{\omega}{\lambda}. \tag{73}$$

Plugging (71) and (73) into (68)

$$\begin{aligned}\int_{\theta_w}^{\overline{\theta_w}}\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)h(s)ds &= \frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{\epsilon_l}{1+\epsilon_l}\theta_w h_w + \frac{z}{pn^{\alpha}}\frac{1}{\sigma}ph_e \\ &= \frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{\epsilon_l}{1+\epsilon_l}\theta_w h_w + \frac{1}{n^{\alpha}}\frac{\beta z^{\sigma}}{\sigma\beta z^{\sigma-1}}h_e \end{aligned}\tag{74}$$

Let ϵ_z be the elasticity of evasion to the corporate tax.

$$\epsilon_z = \frac{\partial z}{\partial T_c'(\cdot)}\frac{T_c'(\cdot)}{z} = \frac{1}{\sigma} \qquad\qquad \frac{\partial z}{\partial T_c'(\cdot)} = \frac{\epsilon_z z}{T_c'(\cdot)} = \frac{1}{\sigma\beta z^{\sigma-1}} \tag{75}$$

We conclude,

$$\int_{\theta_w}^{\overline{\theta_w}}\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)h(s)ds = \frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{\epsilon_l}{1+\epsilon_l}\theta_w h_w + \frac{1}{n^{\alpha}}\frac{\epsilon_z z}{T_c'}T_c'(\cdot)h_e \tag{76}$$

A.5.2 Labor demand optimality condition

Combining the optimality condition (65d) with the implementability condition (69b),

$$T'_n(\cdot)\frac{\omega}{\lambda} = T'_c(\cdot)\frac{\partial z}{\partial n} \tag{77}$$

Deriving (60) we get,

$$0 = p\alpha n^{\alpha-1}\big(1-\beta z^{\sigma}\big) - pn^{\alpha}\beta\sigma z^{\sigma-1}\frac{\partial z}{\partial n} \tag{78}$$

From the implementability condition of labor demand, (69b),

$$\epsilon_n = \frac{\partial n}{\partial(1+T'_n(\cdot))}\frac{(1+T'_n(\cdot))}{n} = -\frac{1}{1-\alpha} \tag{79}$$

Combining the equation (78) above with (69b), (69c) and (75) we can write,

$$\frac{\partial z}{\partial n} = \frac{\frac{\omega}{\lambda}(1+T'_n(\cdot))\big(1-T'_c(\cdot)\big)}{en^{\alpha}}\frac{\epsilon_z z}{T'_c(\cdot)} \tag{80}$$

Hence,

$$\frac{T'_n(\cdot)}{1+T'_n(\cdot)} = \big(1-T'_c(\cdot)\big)\epsilon_z\frac{z}{en^{\alpha}} \tag{81}$$

Or equivalently,

$$\frac{\epsilon_n}{1+\epsilon_n}T'_n(\cdot)\frac{\omega}{\lambda}n = \big(1-T'_c(\cdot)\big)\epsilon_z z \tag{82}$$

Combining (81) and (76)

$$\int_{\underline{\theta_w}}^{\overline{\theta_w}}\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)h(s)ds = \frac{T'_l(\cdot)}{1-T'_l(\cdot)}\frac{\epsilon_l}{1+\epsilon_l}\theta_w h_w + \frac{T'_n(\cdot)}{1+T'_n(\cdot)}\frac{1}{1-T'_c(\cdot)}eh_e \tag{83}$$

In principle, given a schedule $e(\theta_w)$, equations (73), (81) and (83), pin down the optimal marginal tax functions.

A.5.3 Choice function optimality condition

Recall that the Hamiltonian derivative with respect to e is given by:

$$\frac{\partial\mathcal{H}}{\partial e} = -\phi'_e = V_w\frac{\partial h_w(e)}{\partial e} + \frac{\partial V_e}{\partial e}ph_e(e) + V_e p\frac{\partial h_e(e)}{\partial e}, \tag{84}$$

where V_e and V_w are the value functions for the entrepreneurs and the workers, respectively. Recall that, $h_e(\theta_w, e(\theta_w)) = \int_{\underline{\theta_w}}^{\theta_w} g(s, e(\theta_w))ds$, so using the Leibniz rule for integrals and $\frac{dh_e(e)}{d\theta_w} = g(\theta_w, e(\theta_w)) + \int_{\underline{\theta_w}}^{\theta_w}\frac{\partial}{\partial e}g\big(s, e(\theta_w)\big)\cdot\frac{\partial e}{\partial\theta_w}ds$. In addition, $p\frac{\partial h_e}{de} = p\int_{\underline{\theta_w}}^{\theta_w}\frac{\partial}{\partial e}g\big(s, e(\theta_w)\big)ds$, hence, $p\frac{\partial h_e}{de} = \frac{dh_e}{d\theta_w} - g(\theta_w, e)$. Replacing this, and also the derivative of the multiplier function ϕ_e from the derivative of the Hamiltonian with respect to p, in (115), to obtain:

$$\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)+V_eh_e(e)\right]=V_w\frac{\partial h_w(e)}{\partial e}+\frac{\partial V_e}{\partial e}ph_e(e)+V_e\left(\frac{dh_e}{d\theta_w}-g(\theta_w,e)\right).$$

With a little algebra and dividing by u_e':

$$\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\right]\cdot\frac{1}{u_e'}=[V_w-V_e]g(\theta_w,e)\frac{1}{u_e'}+\frac{\partial V_e}{\partial e}ph_e(e)\frac{1}{u_e'}-\frac{dV_e}{d\theta_w}\frac{h_e}{u_e'}. \tag{85}$$

To simplify the term: $\frac{dV_e}{d\theta_w}\frac{h_e}{u_e'}$, recall that: $V_e(\theta_e)=\mathbb{1}u_e(\theta_e)^{\varphi}+\lambda\left(T_n(w_fn)+T_c(\pi-z)\right)$, so the total differential with respect to θ_w is:

$$\begin{aligned}
\frac{dV_e}{d\theta_w}=&\frac{d}{d\theta_w}\left[\mathbb{1}u_e(\theta_e)^{\varphi}+\lambda\left(T_n(w_fn)+T_c(\pi-z)\right)\right]\\
\frac{dV_e}{d\theta_w}=&\left[\mathbb{1}\varphi u_e(\theta_e)^{\varphi-1}u_e'+\lambda T_n'(\cdot)w_f\frac{dn}{de}+\lambda T_c'(\cdot)\frac{d\pi}{de}\right]\cdot\frac{\partial e}{\partial\theta_w}-\lambda T_c'(\cdot)\frac{dz}{d\theta_w}\\
\frac{dV_e}{d\theta_w}=&\left[\mathbb{1}\varphi u_e(\theta_e)^{\varphi-1}u_e'+\lambda T_n'(\cdot)w_f\frac{n}{e}\varepsilon_e^n+\lambda T_c'(\cdot)\frac{d\pi}{de}\right]\cdot p-\lambda T_c'(\cdot)\frac{dz}{d\theta_w}\\
\frac{dV_e}{d\theta_w}\cdot\frac{1}{u_e'}=&\left[\mathbb{1}\varphi u_e(\theta_e)^{\varphi-1}+\lambda T_n'(\cdot)w_f\frac{n}{e}\varepsilon_e^n\frac{1}{u_e'}+\lambda T_c'(\cdot)\frac{d\pi}{de}\frac{1}{u_e'}\right]\cdot p-\lambda T_c'(\cdot)\frac{dz}{d\theta_w}\frac{1}{u_e'}.
\end{aligned}$$

Remember that $u_e(\theta_e)=\pi-T_c(\pi-z)$, hence: $u_e'(\theta_e)=n^\alpha-T_c'(\cdot)n^\alpha=n^\alpha(1-T_c'(\cdot))=n^\alpha(1-\beta z^\sigma)$. Furthermore, $\pi=\theta_en^\alpha-w_fn-T_n(w_fn)$ which implies that:

$$\frac{dV_e}{d\theta_w}\cdot\frac{h_e}{u_e'}=\left[\mathbb{1}\varphi u_e(\theta_e)^{\varphi-1}+\lambda T_n'(\cdot)w_f\frac{n}{e}\varepsilon_e^n\frac{1}{u_e'}+\lambda\frac{T_c'(\cdot)}{(1-T_c'(\cdot))}\right]\cdot h_ep-\lambda\frac{T_c'(\cdot)}{(1-T_c'(\cdot))}\frac{dz}{d\theta_w}\frac{1}{n^\alpha}h_e. \tag{86}$$

And replace this term in equation (85) to obtain:

$$\begin{aligned}
\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\right]\cdot\frac{1}{u_e'}=&[V_w-V_e]g(\theta_w,e)\frac{1}{u_e'}+\frac{\partial V_e}{\partial e}ph_e(e)\frac{1}{u_e'}-\\
&\left(\left[\mathbb{1}\varphi u_e(\theta_e)^{\varphi-1}+\lambda T_n'(\cdot)w_f\frac{n}{e}\varepsilon_e^n\frac{1}{u_e'}+\lambda\frac{T_c'(\cdot)}{(1-T_c'(\cdot))}\right]\cdot h_ep-\lambda\frac{T_c'(\cdot)}{(1-T_c'(\cdot))}\frac{dz}{d\theta_w}\frac{1}{n^\alpha}h_e\right)\leftrightarrow\\
\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\right]\cdot\frac{1}{u_e'}=&[V_w-V_e]g(\theta_w,e)\frac{1}{u_e'}+\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)h_ep-\lambda T_n'(\cdot)w_f\frac{n}{e}\varepsilon_e^n\frac{1}{u_e'}h_ep+\\
&\lambda\frac{T_c'(\cdot)}{(1-T_c'(\cdot))}\frac{dz}{d\theta_w}\frac{1}{n^\alpha}h_e.
\end{aligned} \tag{87}$$

Further, we can simplify the term $\frac{\partial V_e}{\partial p}ph_e(e)$:

$$
\begin{aligned}
\frac{\partial V_e}{\partial p} &= -\lambda\beta z^{\sigma}\frac{\partial z}{\partial p} = -\lambda\beta z^{\sigma}\frac{(1-\beta z^{\sigma})}{p\sigma\beta z^{\sigma-1}} = -\lambda z\frac{(1-\beta z^{\sigma})}{p\sigma} \\
\frac{\partial V_e}{\partial p}ph_e &= -\lambda z\frac{(1-\beta z^{\sigma})}{p\sigma}ph_e = -\lambda\varepsilon^{z}_{T'_c}z(1-T'_c)h_e \\
\frac{\partial V_e}{\partial p}ph_e\frac{1}{u'_e} &= -\lambda\varepsilon^{z}_{T'_c}z(1-T'_c)\frac{1}{u'_e}h_e = -\lambda\varepsilon^{z}_{T'_c}\frac{z}{n^{\alpha}}h_e.
\end{aligned}
$$

the last line multiplying by $\frac{1}{u'_e}$. Notice that the term that we obtained $-\lambda\varepsilon^{z}_{T'_c}\frac{z}{n^{\alpha}}h_e$ is the same for the effect of the increase in the marginal rate of the corporate tax, multiplied by λ. Recall that equating the effect of an increase in the marginal corporate and payroll tax gives us: $\frac{T'_n(\cdot)}{1+T'_n(\cdot)} = (1-T'_c(\cdot))\varepsilon^{z}_{T'_c}\frac{z}{en^{\alpha}}$, which implies the following: $-\lambda\varepsilon^{z}_{T'_c}\frac{z}{n^{\alpha}}h_e = -\lambda\frac{T'_n(\cdot)}{1+T'_n(\cdot)}\frac{e}{1-T'_c(\cdot)}h_e$. Moreover:

$$
\begin{aligned}
\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\cdot\frac{1}{u'_e}\right] =& \frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\right]\cdot\frac{1}{u'_e} + \frac{d}{d\theta_w}\left[\frac{1}{u'_e}\right]\cdot\frac{\partial V_e}{\partial p}ph_e(e) \\
\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\right]\cdot\frac{1}{u'_e} =& \frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\cdot\frac{1}{u'_e}\right] - \frac{d}{d\theta_w}\left[\frac{1}{u'_e}\right]\cdot\frac{\partial V_e}{\partial p}ph_e(e) \\
\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\right]\cdot\frac{1}{u'_e} =& \underbrace{\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\cdot\frac{1}{u'_e}\right]}_{\text{A}} - \underbrace{\frac{du'_e}{d\theta_w}\cdot\frac{1}{{u'_e}^2}u'_e\lambda\varepsilon^{z}_{T'_c}\frac{z}{n^{\alpha}}h_e}_{\text{B}}
\end{aligned}
$$

Part A can be expressed as:

$$
\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\cdot\frac{1}{u'_e}\right] = \frac{d}{d\theta_w}\left[-\lambda\varepsilon^{z}_{T'_c}\frac{z}{n^{\alpha}}h_e\right],
$$

while part B is:

$$
\begin{aligned}
\frac{du'_e}{d\theta_w}\cdot\frac{1}{{u'_e}^2}u'_e\lambda\varepsilon^{z}_{T'_c}\frac{z}{n^{\alpha}}h_e =& \frac{d(n^{\alpha}(1-\beta z^{\sigma}))}{d\theta_w}\cdot\frac{1}{n^{\alpha}(1-\beta z^{\sigma})}\lambda\varepsilon^{z}_{T'_c}\frac{z}{n^{\alpha}}h_e \\
=& \left(\alpha n^{\alpha-1}(1-\beta z^{\sigma})\frac{dn}{de}\frac{de}{d\theta_w} - n^{\alpha}\sigma\beta z^{\sigma-1}\frac{dz}{d\theta_w}\right)\frac{1}{n^{\alpha}(1-\beta z^{\sigma})}\lambda\varepsilon^{z}_{T'_c}\frac{z}{n^{\alpha}}h_e \\
=& \left(\lambda\varepsilon^{z}_{T'_c}\frac{z}{n^{\alpha}}\frac{\alpha}{n}\frac{dn}{de}ph_e - \lambda\frac{1}{n^{\alpha}}\frac{dz}{d\theta_w}\frac{\beta z^{\sigma}}{1-\beta z^{\sigma}}h_e\right) \\
=& \left(\lambda\frac{T'_n(\cdot)}{1+T'_n(\cdot)}\frac{e}{1-T'_c(\cdot)}\frac{\alpha}{n}\frac{dn}{de}ph_e - \frac{\lambda}{n^{\alpha}}\frac{dz}{d\theta_w}\frac{T'_c(\cdot)}{1-T'_c(\cdot)}h_e\right) \\
=& \left(\lambda\frac{T'_n(\cdot)}{1+T'_n(\cdot)}\frac{1}{1-T'_c(\cdot)}\alpha\varepsilon^{n}_{e}ph_e - \frac{\lambda}{n^{\alpha}}\frac{dz}{d\theta_w}\frac{T'_c(\cdot)}{1-T'_c(\cdot)}h_e\right).
\end{aligned}
$$

With these simplifications we can get:

$$
\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(e)\right]\cdot\frac{1}{u'_e} = \frac{d}{d\theta_w}\left[-\lambda\varepsilon^{z}_{T'_c}\frac{z}{n^{\alpha}}h_e\right] - \left(\lambda\frac{T'_n(\cdot)}{1+T'_n(\cdot)}\frac{1}{1-T'_c(\cdot)}\alpha\varepsilon^{n}_{e}ph_e - \frac{\lambda}{n^{\alpha}}\frac{dz}{d\theta_w}\frac{T'_c(\cdot)}{1-T'_c(\cdot)}h_e\right).
$$

We can equate this expression with (87) and obtain:

$$\frac{d}{d\theta_w}\left[-\lambda\varepsilon^z_{T'_c}\frac{z}{n^\alpha}h_e\right]-\left(\lambda\frac{T'_n(\cdot)}{1+T'_n(\cdot)}\frac{1}{1-T'_c(\cdot)}\alpha\varepsilon^n_e p h_e-\frac{\lambda}{n^\alpha}\frac{dz}{d\theta_w}\frac{T'_c(\cdot)}{1-T'_c(\cdot)}h_e\right)=$$
$$[V_w-V_e]g(\theta_w,e)\frac{1}{u'_e}+\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)h_e p-\lambda T'_n(\cdot)w_f\frac{n}{e}\varepsilon^n_e\frac{1}{u'_e}h_e p+\lambda\frac{T'_c(\cdot)}{(1-T'_c(\cdot))}\frac{dz}{d\theta_w}\frac{1}{n^\alpha}h_e\leftrightarrow$$
$$\frac{d}{d\theta_w}\left[-\lambda\varepsilon^z_{T'_c}\frac{z}{n^\alpha}h_e\right]=[V_w-V_e]g(\theta_w,e)\frac{1}{u'_e}+\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)h_e p,$$

and when we integrate the previous from an specific productivity:

$$\frac{d}{d\theta_w}\left[-\lambda\varepsilon^z_{T'_c}\frac{z}{n^\alpha}h_e\right]=[V_w-V_e]g(\theta_w,e)\frac{1}{u'_e}+\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)h_e p,$$
$$\int_{\theta_e}^{\overline{\theta_e}}\frac{d}{d\theta_w}\left[-\lambda\varepsilon^z_{T'_c}\frac{z}{n^\alpha}h_e\right]ds=\int_{\theta_e}^{\overline{\theta_e}}[V_w-V_e]g(\theta_w,s)\frac{1}{u'_e}ds+\int_{\theta_e}^{\overline{\theta_e}}\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)h_e(\theta_w,s)pds.$$

Moreover, the left part of the equation is:

$$\int_{\theta_e}^{\overline{\theta_e}}\frac{d}{d\theta_w}\left[-\lambda\varepsilon^z_{T'_c}\frac{z}{n^\alpha}h_e\right]ds=-\lambda\varepsilon^z_{T'_c}\frac{z}{n^\alpha}h_e(\theta_w,\overline{\theta_e})+\lambda\varepsilon^z_{T'_c}\frac{z}{n^\alpha}h_e=\lambda\varepsilon^z_{T'_c}\frac{z}{n^\alpha}h_e,$$

as there is 0 marginal taxation at the top of the distribution. Also, recall that $V_w(\theta_w)=\mathbb{1}u_w^\varphi+\lambda T_l(\cdot)$, and $V_e(\theta_e)=\mathbb{1}u_e(\theta_e)^\varphi+\lambda\left(T_n(\cdot)+T_c(\cdot)\right)$, hence: $[V_w-V_e]=\lambda\big[T_l(\cdot)-T_c(\cdot)-T_n(\cdot)\big]$. Finally, we obtain:

$$\lambda\varepsilon^z_{T'_c}\frac{z}{n^\alpha}h_e(\theta_w,e)=\int_{\theta_e}^{\overline{\theta_e}}\lambda\big[T_l(\cdot)-T_c(\cdot)-T_n(\cdot)\big]g(\theta_w,s)\frac{1}{u'_e}ds+\int_{\theta_e}^{\overline{\theta_e}}\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)h_e(\theta_w,s)pds\leftrightarrow$$
$$\int_{\theta_e}^{\overline{\theta_e}}\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)h_e(\theta_w,s)pds=\lambda\varepsilon^z_{T'_c}\frac{z}{n^\alpha}h_e-\int_{\theta_e}^{\overline{\theta_e}}\lambda\big[T_l(\cdot)-T_c(\cdot)-T_n(\cdot)\big]g(\theta_w,s)\frac{1}{u'_e}ds \tag{88}$$

A.6 Derivation of the Optimal Tax Formulas With Informality

With informality, the planner' problem is as follows:

$$\max_{l,n,p} \int_{\underline{\theta_w}}^{\overline{\theta_w}} \mathbb{1} u_w^{\varphi} h \, d\theta + (1-\mathbb{1}) u_w(\underline{\theta_w}) h(\underline{\theta_w}), \tag{89a}$$

$$\text{s.t. } u_w' = \frac{\chi}{\theta_w} l^{1+\psi} \tag{89b}$$

$$Y' = \left\{ en^{\alpha} - \frac{\delta}{1+\gamma} n_i^{1+\gamma} - \frac{\beta}{1+\sigma} z^{1+\sigma} - u_w \right\} ph_e - \left\{ u_w + \frac{\chi}{1+\psi} l^{1+\psi} + \frac{\kappa}{1+\rho} (\theta_w l_i)^{1+\rho} \right\} h_w \tag{89c}$$

$$L_f' = \theta_w (l - l_i) h_w - (n - n_i) ph_e \tag{89d}$$

$$L_i' = \theta_w l_i h_w - n_i ph_e \tag{89e}$$

$$e' = p, \qquad w_i' = 0, \text{ and the boundary conditions} \tag{89f}$$

$$Y(\underline{\theta_w}) = L_f(\underline{\theta_w}) = L_i(\underline{\theta_w}) = 0, \quad Y(\overline{\theta_w}) \geq G, \quad L_f(\overline{\theta_w}) \geq 0, \quad L_i(\overline{\theta_w}) \geq 0. \tag{89g}$$

And the transversality conditions.

Where $z(l, n, p; \theta_w)$, $n_i(l, n, p, e; \theta_w)$ and $l_i(l; \theta_w)$ are implicitly defined, respectively, by,

$$\frac{\chi}{\theta_w} l^{1+\psi} = pn^{\alpha}(1 - \beta z^{\sigma}), \tag{90a}$$

$$\delta n_i^{\gamma} = \left(\alpha e n^{\alpha-1} - w_i \right), \tag{90b}$$

$$\theta_w w_i - \chi l^{\psi} = \kappa \theta_w^{1+\rho} l_i^{\rho}. \tag{90c}$$

Let $\mu, \lambda, \omega_i, \omega_f, \phi_e,$ and ϕ_w be the multiplier functions associated with $u_w, Y, L_i, L_f, e,$ and w_i, respectively. The Hamiltonian, for this problem is:

$$\begin{aligned} \mathcal{H} =& \mathbb{1} u_w^{\varphi} h + \mu \frac{\chi}{\theta_w} l^{1+\psi} + \omega_f [\theta_w l h_w - nph_e] + (\omega_i - \omega_f)[\theta_w l_i h_w - n_i ph_e] + \phi_e[p] + \phi_w[0] + \\ & \lambda \left\{ en^{\alpha} - \frac{\delta}{1+\gamma} n_i^{1+\gamma} - \frac{\beta}{1+\sigma} z^{1+\sigma} - u_w \right\} ph_e - \lambda \left\{ u_w + \frac{\chi}{1+\psi} l^{1+\psi} + \frac{\kappa}{1+\rho} (\theta_w l_i)^{1+\rho} \right\} h_w. \end{aligned} \tag{91}$$

Where the state variables are u_w, Y, L_i, L_f, e and w_i, the controls are l, n, p and l_i, n_i, z that are defined in terms of the later by the equations (90). We define the value for the planner of a worker, adjusted by the surplus of goods it gives to the planner, and analogously for an entrepreneur:

$$V_w(\theta_w) = \mathbb{1} u_w^{\varphi} + \left[\theta_w l \omega_f + (\omega_i - \omega_f) \theta_w l_i - \lambda \left(u_w + \frac{\chi l^{1+\psi}}{1+\psi} + \frac{\kappa (\theta_w l_i)^{1+\rho}}{1+\rho} \right) \right], \tag{92a}$$

$$\hat{V}_e(\theta_e) = \mathbb{1} u_e^{\varphi}(\theta_e) + \lambda \theta_e n^{\alpha}(\theta_e) - \lambda \frac{\beta z_e^{1+\sigma}(\theta_e)}{1+\sigma} - \omega_f n(\theta_e) - (\omega_i - \omega_f) n_i(\theta_e) - \lambda u_e(\theta_e) - \lambda \frac{\delta n_i^{1+\gamma}(\theta_e)}{1+\gamma}. \tag{92b}$$

The utility of workers and entrepreneurs is defined by,

$$u_w = c_w - \frac{\chi}{1+\psi} l^{1+\psi} = \theta_w(w_f(l - l_i) + w_i l_i) - \frac{\kappa}{1+\rho}(\theta_w l_i)^{1+\rho} - T_l(\cdot) - \frac{\chi}{1+\psi} l^{1+\psi}. \tag{93}$$

From the equation above, we can write the labor tax as,

$$\lambda T_l(\cdot) = \theta_w(\omega_f(l - l_i) + \omega_i l_i) - \lambda\bigg(u_w + \frac{\chi l^{1+\psi}}{1+\psi} + \frac{\kappa(\theta_w l_i)^{1+\rho}}{1+\rho}\bigg). \tag{94}$$

Similarly for entrepreneurs,

$$u_e = c_e = \theta_e n^\alpha - w_i n_i - w_f(n - n_i) - T_n(\cdot) - T_c(\cdot) - \frac{\delta}{1+\gamma} n_i^{1+\gamma} - \frac{\beta}{1+\sigma} z^{1+\sigma}, \tag{95}$$

and,

$$\lambda\left(T_n(\cdot) + T_c(\cdot)\right) = -(\omega_i n_i + \omega_f(n - n_i)) + \lambda\left[\theta_e n^\alpha - \frac{\delta}{1+\gamma} n_i^{1+\gamma} - \frac{\beta}{1+\sigma} z^{1+\sigma} - u_e\right]. \tag{96}$$

So, the value for the planner of the worker and entrepreneur are,

$$\begin{aligned} V_w(\theta_w) &= \mathbb{1} u_w^\varphi(\theta_w) + \lambda T_l(\cdot), \\ \hat{V}_e(\theta_e) &= \mathbb{1} u_e^\varphi(\theta_e) + \lambda\big(T_n(\cdot) + T_c(\cdot)\big), \end{aligned} \tag{97a}$$

and in terms of θ_w set $V_e(\theta_w) = \hat{V}_e\big(e(\theta_w)\big)$ as:

$$V_e(\theta_w) = \mathbb{1} u_w^\varphi + \lambda e n^\alpha - \lambda \frac{\beta z^{1+\sigma}}{1+\sigma} - n_i(\omega_i - \omega_f) - \omega_f n - \lambda u_w - \lambda \frac{\delta n_i^{1+\gamma}}{1+\gamma}. \tag{97b}$$

The Hamiltionan $\mathcal{H}$ (equation 91) can be rewritten using the planner valuations of the agents utility and output (equations 97) as:

$$\mathcal{H} = V_w h_w + V_e p h_e + \mu \frac{\chi l^{1+\psi}}{\theta_w} + \phi_e[p] + \phi_w[0]. \tag{98}$$

The optimality conditions for the control variables of this problem are:

$$\begin{aligned} \{l\} : 0 =& \mu(1+\psi)\frac{\chi}{\theta_w} l^\psi + \omega_f \theta_w h_w + (\omega_i - \omega_f)\left[\theta_w \frac{\partial l_i}{\partial l} h_w - \frac{\partial n_i}{\partial l} p h_e\right] \\ &+ \lambda\left[-\delta n_i^\gamma \frac{\partial n_i}{\partial l} - \beta z^\sigma \frac{\partial z}{\partial l}\right] p h_e - \lambda\left[\chi l^\psi + \kappa \theta_w^{1+\rho} l_i^\rho \frac{\partial l_i}{\partial l}\right] h_w, \end{aligned} \tag{99a}$$

$$\{n\} : 0 = -\omega_f p h_e - (\omega_i - \omega_f)\frac{\partial n_i}{\partial n} p h_e + \lambda\left[\alpha e n^{\alpha-1} - \delta n_i^\gamma \frac{\partial n_i}{\partial n} - \beta z^\sigma \frac{\partial z}{\partial n}\right] p h_e, \tag{99b}$$

$$\begin{aligned} \{p\} : 0 =& \mathbb{1} u_w^\varphi h_e - \omega_f n h_e + (\omega_i - \omega_f)\left[-n_i h_e - p h_e \frac{\partial n_i}{\partial p}\right] + \phi_e - \lambda\left[\delta n_i^\gamma \frac{\partial n_i}{\partial p} + \beta z^\sigma \frac{\partial z}{\partial p}\right] p h_e \\ &+ \lambda\left[e n^\alpha - \frac{\delta}{1+\gamma} n_i^{1+\gamma} - \frac{\beta}{1+\sigma} z^{1+\sigma} - u_w\right] h_e, \end{aligned} \tag{99c}$$

and for the state variables,

$$
\begin{aligned}
\{e\}: -\phi_e' =& \frac{\partial h_e}{\partial e}\left((\mathbb{1}u_w^{\varphi} - \lambda u_w) - \omega_f n - (\omega_i - \omega_f)n_i + \lambda\left[en^{\alpha} - \frac{\delta}{1+\gamma}n_i^{1+\gamma} - \frac{\beta}{1+\sigma}z^{1+\sigma}\right]\right)p \\
&+ \frac{\partial h_w}{\partial e}\left((\mathbb{1}u_w^{\varphi} - \lambda u_w) + \omega_f l\theta_w + (\omega_i - \omega_f)l_i\theta_w - \lambda\left[\frac{\chi}{1+\psi}l^{1+\psi} + \frac{\kappa}{1+\rho}(\theta_w l_i)^{1+\rho}\right]\right) \\
&+ \lambda n^{\alpha} p h_e - \frac{\partial n_i}{\partial e}\left[\lambda\delta n_i^{\gamma} + (\omega_i - \omega_f)\right]ph_e \qquad (99d) \\
\{u_w\}: -\mu' =& \mathbb{1}(\varphi u_w^{\varphi-1}h) - \lambda p h_e - \lambda h_w = h(\mathbb{1}\varphi u_w^{\varphi-1} - \lambda), \qquad (99e) \\
\{w_i\}: -\phi_w' =& (\omega_i - \omega_f)\left[\theta_w h_w \frac{\partial l_i}{\partial w_i} - ph_e\frac{\partial n_i}{\partial w_i}\right] - \lambda\left[\delta n_i^{\gamma}\frac{\partial n_i}{\partial w_i}ph_e + \kappa\theta_w^{1+\rho}l_i^{\rho}\frac{\partial l_i}{\partial w_i}h_w\right], \qquad (99f) \\
\{Y\}: -\lambda' =& 0, \qquad (99g) \\
\{L_i\}: -\omega_i' =& 0, \qquad (99h) \\
\{L_f\}: -\omega_f' =& 0. \qquad (99i)
\end{aligned}
$$

These equations can be rewritten in terms of the values of worker and entrepreneur. Neither of Y, L_i and L_f appears on the Hamiltonian $\mathcal{H}$, hence the state optimality conditions yield:

$$\lambda' = \omega_f' = \omega_i' = 0. \qquad (100a)$$

which imply those multiplier functions are constant.

The optimality conditions for the workers utility profile u_w is:

$$\frac{\partial \mathcal{H}}{\partial u_w} = -\mu' = h(\mathbb{1}\varphi u_w^{\varphi-1} - \lambda). \qquad (100b)$$

Now, notice that the defined distribution functions $h_w(\theta_w), h_e(\theta_w)$ all depend on e but only through the value of $e(\theta_w)$ and not $e'(\theta_w)$. Hence, using the short version of the Hamiltonian given by equation (98)

$$\frac{\partial \mathcal{H}}{\partial e} = -\phi_e' = V_w\frac{\partial h_w(e)}{\partial e} + \frac{\partial V_e}{\partial e}ph_e(e) + V_e p\frac{\partial h_e(e)}{\partial e}. \qquad (100c)$$

The optimality condition with respect to labor allocation to firms n is:

$$\frac{\partial \mathcal{H}}{\partial n} = \left(-\omega_f - (\omega_i - \omega_f)\frac{\partial n_i}{\partial n} + \lambda\left[e\alpha n^{\alpha-1} - \delta n_i^{\gamma}\frac{\partial n_i}{\partial n} - \beta z^{\sigma}\frac{\partial z}{\partial n}\right]\right)ph_e(e) = 0. \qquad (100d)$$

With respect to the labor supply $\frac{\partial \mathcal{H}}{\partial l}$:

$$\frac{\partial \mathcal{H}}{\partial l} = \frac{\mu}{\theta_w}\chi(1+\psi)l^{\psi} + h_w\frac{\partial V_w}{\partial l} + ph_e(e)\frac{\partial V_e}{\partial l} = 0, \qquad (100e)$$

where, $\frac{\partial V_w}{\partial l} = \omega_f\theta_w - \lambda\left[\chi l^{\psi} + \kappa\theta_w^{1+\rho}l_i^{\rho}\frac{\partial l_i}{\partial l}\right]$ and $\frac{\partial V_e}{\partial l} = -\lambda\left[\delta n_i^{\gamma}\frac{\partial n_i}{\partial l} + \beta z^{\sigma}\frac{\partial z}{\partial l}\right]$.

The optimality condition for the derivative of the choice function $\frac{\partial \mathcal{H}}{\partial p}$ is:

$$\frac{\partial V_e}{\partial p} p h_e(e) + V_e h_e(e) + \phi_e = 0, \tag{100f}$$

where, $\frac{\partial V_e}{\partial p} = -\lambda \beta z^\sigma \frac{\partial z}{\partial p}$. The optimality condition regarding the informal wage w_i is:

$$\frac{\partial \mathcal{H}}{\partial w_i} = -\phi'_w = (\omega_i - \omega_f)\left[\theta_w h_w \frac{\partial l_i}{\partial w_i} - p h_e \frac{\partial n_i}{\partial w_i}\right] - \lambda\left[\delta n_i^\gamma \frac{\partial n_i}{\partial w_i} p h_e + \kappa \theta_w^{1+\rho} l_i^\rho \frac{\partial l_i}{\partial w_i} h_w\right]$$

A.6.1 Labor supply optimality condition

From the implementation of the worker's problem we have

$$\theta_w w_f (1 - T'_l(\cdot)) - \chi l^\psi = 0 \leftrightarrow \frac{\theta_w \omega_f}{\lambda} - \chi l^\psi = T'_l(\cdot)\frac{\theta_w \omega_f}{\lambda},$$

hence $\frac{\lambda \chi l^\psi}{\theta_w \omega_f} = 1 - T'_l(\cdot)$. Also $\frac{1}{\varepsilon_l} = \psi$, where $\varepsilon_l = \frac{\partial l}{\partial(1 - T'_l(\cdot))}\frac{(1-T'_l)}{l}$ is the price elasticity of labor. Hence we have:

$$\left[1 - T'_l(\cdot)\right]\left[1 + \frac{1}{\varepsilon_l}\right] = \frac{\lambda \chi l^\psi}{\theta_w \omega_f}(1 + \psi). \tag{101}$$

Since μ is the multiplier function on the labor IC constraint, we have $\mu(\overline{\theta_w}) = 0$ and hence from the optimality condition for u_w, equation (100b), we get

$$\mu(\theta_w) = \int_{\theta_w}^{\overline{\theta_w}} -\frac{d\mu(s)}{ds} ds = \int_{\theta_w}^{\overline{\theta_w}} (\mathbb{1}\varphi u_w^{\varphi - 1} - \lambda) h(s) ds. \tag{102}$$

Divide the optimality condition for labor supply equation (99a) by ω_f, replace from equation (101) and divide by $\left[1 - T'_l(\cdot)\right]\left[1 + \frac{1}{\varepsilon_l}\right]$, then use equation (102) to obtain:

$$\begin{aligned}
-\frac{\mu}{\lambda} =& \frac{T'_l(\cdot)}{1 - T'_l(\cdot)}\frac{\varepsilon_l}{1+\varepsilon_l}\theta_w h_w - \frac{\varepsilon_l}{1+\varepsilon_l}\frac{1}{1 - T'_l(\cdot)}\frac{\lambda}{\omega_f}\left[\left(\delta n_i^\gamma \frac{\partial n_i}{\partial l} + \beta z^\sigma \frac{\partial z}{\partial l}\right) p h_e + \right. \\
& \left. + \kappa \theta_w^{1+\rho} l_i^\rho \frac{\partial l_i}{\partial l} h_w\right] + \frac{\omega_i - \omega_f}{\omega_f}\frac{\varepsilon_l}{1+\varepsilon_l}\frac{1}{1 - T'_l(\cdot)}\left(\theta_w \frac{\partial l_i}{\partial l} h_w - \frac{\partial n_i}{\partial l} p h_e\right) \leftrightarrow \\
\int_{\theta_w}^{\overline{\theta_w}}\left(1 - \mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right) h(s) ds =& \frac{T'_l(\cdot)}{1 - T'_l(\cdot)}\frac{\varepsilon_l}{1+\varepsilon_l}\theta_w h_w - \frac{\varepsilon_l}{1+\varepsilon_l}\frac{1}{1 - T'_l(\cdot)}\frac{\lambda}{\omega_f}\left[\left(\delta n_i^\gamma \frac{\partial n_i}{\partial l} + \beta z^\sigma \frac{\partial z}{\partial l}\right) p h_e + \right. \\
& \left. + \kappa \theta_w^{1+\rho} l_i^\rho \frac{\partial l_i}{\partial l} h_w\right] + \frac{\omega_i - \omega_f}{\omega_f}\frac{\varepsilon_l}{1+\varepsilon_l}\frac{1}{1 - T'_l(\cdot)}\left(\theta_w \frac{\partial l_i}{\partial l} h_w - \frac{\partial n_i}{\partial l} p h_e\right).
\end{aligned} \tag{103}$$

Taking into account the derivation of the problem without informality, and that $z(\cdot)$ is the same when there isn't informality, we can follow the previous appendix and equation (103) can be rewritten in terms of the price elasticity of labor (ε_l) and the elasticity of evasion to the corporate tax (ε_z) as,

$$\begin{aligned}\int_{\theta_w}^{\overline{\theta_w}}\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)h(s)ds =&\frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{\varepsilon_l}{1+\varepsilon_l}\theta_w h_w+\frac{\varepsilon_z z}{n^\alpha}h_e\\ &-\frac{\varepsilon_l}{1+\varepsilon_l}\frac{1}{1-T_l'(\cdot)}\frac{\lambda}{\omega_f}\left[\delta n_i^\gamma\frac{\partial n_i}{\partial l}ph_e+\kappa\theta_w^{1+\rho}l_i^\rho\frac{\partial l_i}{\partial l}h_w\right]\\ &+\frac{\omega_i-\omega_f}{\omega_f}\frac{\varepsilon_l}{1+\varepsilon_l}\frac{1}{1-T_l'(\cdot)}\left(\theta_w\frac{\partial l_i}{\partial l}h_w-\frac{\partial n_i}{\partial l}ph_e\right),\end{aligned}\tag{104}$$

rearranging terms, last equation is equivalent to,

$$\begin{aligned}\int_{\theta_w}^{\overline{\theta_w}}\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)h(s)ds =&\frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{\varepsilon_l}{1+\varepsilon_l}\theta_w h_w+\frac{\varepsilon_z z}{n^\alpha}h_e\\ &-\frac{\varepsilon_l}{1+\varepsilon_l}\frac{1}{1-T_l'(\cdot)}\frac{\partial n_i}{\partial l}\frac{1}{\omega_f}\underbrace{\left[\lambda\delta n_i^\gamma+\omega_i-\omega_f\right]}_{\text{A}}ph_e\\ &+\frac{\varepsilon_l}{1+\varepsilon_l}\frac{1}{1-T_l'(\cdot)}\frac{\partial l_i}{\partial l}\frac{1}{\omega_f}\underbrace{\left[\omega_i-\omega_f-\lambda\kappa(\theta_w l_i)^\rho\right]}_{\text{B}}\theta_w h_w.\end{aligned}\tag{105}$$

For part A, from the implementability FOC for firms we have, as long as $T_c'(\cdot)\neq 1$

$$\begin{aligned}\left(-w_i+w_f(1+T_n'(\cdot))-\delta n_i^\gamma\right)\left(1-T_c'(\cdot)\right)=&0\leftrightarrow\\ -\omega_i+\omega_f(1+T_n'(\cdot))=&\lambda\delta n_i^\gamma\leftrightarrow\\ \omega_f T_n'(\cdot)=&\lambda\delta n_i^\gamma+\omega_i-\omega_f.\end{aligned}\tag{106}$$

While for part B, use the optimality of the informal market of the workers,

$$\begin{aligned}\theta_w\left(\frac{\omega_i}{\lambda}-\frac{\omega_f}{\lambda}(1+T_l'(\cdot))\right)-\kappa\theta_w^{1+\rho}l_i^\rho=&0\leftrightarrow\\ (\omega_i-\omega_f)-\lambda\kappa(\theta_w l_i)^\rho=&-\omega_f T_l'(\cdot).\end{aligned}\tag{107}$$

Therefore, (105) is:

$$\begin{aligned}\int_{\theta_w}^{\overline{\theta_w}}\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)h(s)ds =&\frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{\varepsilon_l}{1+\varepsilon_l}\theta_w h_w+\frac{\varepsilon_z z}{n^\alpha}h_e\\ &-\frac{\varepsilon_l}{1+\varepsilon_l}\frac{T_n'(\cdot)}{1-T_l'(\cdot)}\frac{\partial n_i}{\partial l}ph_e\\ &-\frac{\varepsilon_l}{1+\varepsilon_l}\frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{\partial l_i}{\partial l}\theta_w h_w.\end{aligned}\tag{108}$$

Now, let $\varepsilon_{l_i} = \frac{\partial l_i}{\partial(1-T_l')}\frac{(1-T_l')}{l_i}$. Notice tha equation (90c) comes from the workers' f.o.c. w.r.t l_i replacing the term $w_f(1-T_l'(\cdot))$ by the f.o.c. w.r.t l. Hence, the use of elasticities w.r.t $(1-T_l'(\cdot))$:

$$\frac{\partial l_i}{\partial l} = \frac{\varepsilon_{l_i}}{\varepsilon_l}\cdot\frac{l_i}{l},$$

and we can write:

$$\int_{\theta_w}^{\overline{\theta_w}}\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)h(s)ds = \frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{1}{1+\varepsilon_l}\left(\varepsilon_l-\varepsilon_{l_i}\frac{l_i}{l}\right)\theta_w h_w + \frac{\varepsilon_z z}{n^\alpha}h_e - \frac{\varepsilon_l}{1+\varepsilon_l}\frac{T_n'(\cdot)}{1-T_l'(\cdot)}\frac{\partial n_i}{\partial l}ph_e. \tag{109}$$

Recall that n_i is defined implicitly by equation (90b). Note that in this case, the expression doesn't depends on l, which means that $\frac{\partial n_i}{\partial l}=0$. Which implies that (109) is:

$$\int_{\theta_w}^{\overline{\theta_w}}\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)h(s)ds = \frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{1}{1+\varepsilon_l}\left(\varepsilon_l-\varepsilon_{l_i}\frac{l_i}{l}\right)\theta_w h_w + \frac{\varepsilon_z z}{n^\alpha}h_e. \tag{110}$$

A.6.2 Labour demand optimality condition

The derivative of the Hamiltonian respect to n (equation 100d), can be expressed as,

$$\left[-(\omega_i-\omega_f)-\lambda\delta n_i^\gamma\right]\frac{\partial n_i}{\partial n} + \underbrace{\left[\lambda e\alpha n^{\alpha-1}-\omega_f-\lambda\beta z^\sigma\frac{\partial z}{\partial n}\right]}_{\text{C}} = 0.$$

For part C note that $\frac{\partial z}{\partial n}$ is:

$$\frac{\partial z}{\partial n} = \alpha\frac{\varepsilon_z z}{n}\frac{1-T_c'(\cdot)}{T_c'(\cdot)},$$

we can replace the value of $\alpha = \frac{\omega_f}{\lambda}(1+T_n'(\cdot))\frac{n^{1-\alpha}}{e}$, from the FOC of the entrepreneurs respect to n:

$$\frac{\partial z}{\partial n} = \frac{\omega_f}{\lambda}(1+T_n'(\cdot))\frac{1}{e}\frac{\varepsilon_z z}{n^\alpha}\frac{1-T_c'(\cdot)}{T_c'(\cdot)}.$$

Using the previous definition for the derivative of z with respect to n, and also from the first order condition of the entrepreneurs with respect to n the fact that: $\omega_f T_n'(\cdot) = \lambda e\alpha n^{\alpha-1}-\omega_f$, we can simplify D as:

$$\left[\lambda e\alpha n^{\alpha-1}-\omega_f-\lambda\beta z^\sigma\frac{\partial z}{\partial n}\right] = \omega_f T_n'(\cdot) - \omega_f(1+T_n'(\cdot))\frac{1}{e}\frac{\varepsilon_z z}{n^\alpha}(1-T_c'(\cdot)). \tag{111}$$

In addition, notice that with equation (106), we can rewrite the derivative of the Hamiltonian with respect to n as:

$$
\begin{aligned}
-\omega_f T_n'(\cdot)\frac{\partial n_i}{\partial n} + \omega_f T_n'(\cdot) - \omega_f(1+T_n'(\cdot))\frac{1}{e}\frac{\varepsilon_z z}{n^\alpha}(1-T_c'(\cdot)) =& 0 \\
\omega_f T_n'(\cdot)\left(1-\frac{\partial n_i}{\partial n}\right) =& \omega_f(1+T_n'(\cdot))\frac{1}{e}\frac{\varepsilon_z z}{n^\alpha}(1-T_c'(\cdot)).
\end{aligned} \tag{112}
$$

With the last equation we can find the value of $\frac{\varepsilon_z z}{n^\alpha}$:

$$
\frac{\varepsilon_z z}{n^\alpha} = \frac{eT_n'(\cdot)}{(1+T_n'(\cdot))(1-T_c'(\cdot))}\left(1-\frac{\partial n_i}{\partial n}\right), \tag{113}
$$

witch is the same as in equation (110), hence replacing it:

$$
\int_{\theta_w}^{\overline{\theta_w}}\left(1-\mathbb{1}\frac{\varphi u_w^{\varphi-1}}{\lambda}\right)h(s)ds = \frac{T_l'(\cdot)}{1-T_l'(\cdot)}\frac{1}{1+\varepsilon_l}\left(\varepsilon_l-\varepsilon_{l_i}\frac{l_i}{l}\right)\theta_w h_w + \frac{T_n'(\cdot)}{(1+T_n'(\cdot))(1-T_c'(\cdot))}\left(1-\frac{\partial n_i}{\partial n}\right)eh_e. \tag{114}
$$

A.6.3 Choice function optimality condition

Recall that the Hamiltonian derivative with respect to e is given by:

$$
\frac{\partial \mathcal{H}}{\partial e} = -\phi_e' = V_w\frac{\partial h_w(\theta_w,e)}{\partial e} + \frac{\partial V_e}{\partial e}ph_e(\theta_w,e) + V_e p\frac{\partial h_e(\theta_w,e)}{\partial e}, \tag{115}
$$

where V_e and V_w are the value functions for the entrepreneurs and the workers, respectively. Recall that, $h_e(\theta_w,e(\theta_w)) = \int_{\underline{\theta_w}}^{\theta_w} g(s,e(\theta_w))ds$, so using the Leibnitz rule for integrals and $\frac{dh_e(\theta_w,e)}{d\theta_w} = g(\theta_w,e(\theta_w)) + \int_{\underline{\theta_w}}^{\theta_w}\frac{\partial}{\partial e}g\big(s,e(\theta_w)\big)\cdot\frac{\partial e}{\partial\theta_w}ds$. In addition, $p\frac{\partial h_e(\theta_w,e)}{de} = p\int_{\underline{\theta_w}}^{\theta_w}\frac{\partial}{\partial e}g\big(s,e(\theta_w)\big)ds$, hence, $p\frac{\partial h_e(\theta_w,e)}{de} = \frac{dh_e(\theta_w,e)}{d\theta_w} - g(\theta_w,e)$. Replacing this, and also the derivative of the multiplier function ϕ_e from the derivative of the Hamiltonian with respect to p, in (115), we obtain:

$$
\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(\theta_w,e) + V_e h_e(\theta_w,e)\right] = V_w\frac{\partial h_w(\theta_w,e)}{\partial e} + \frac{\partial V_e}{\partial e}ph_e(\theta_w,e) + V_e\left(\frac{dh_e(\theta_w,e)}{d\theta_w} - g(\theta_w,e)\right).
$$

With a little algebra and dividing by u_e':

$$
\frac{d}{d\theta_w}\left[\frac{\partial V_e}{\partial p}ph_e(\theta_w,e)\right]\cdot\frac{1}{u_e'} = \big[V_w - V_e\big]g(\theta_w,e)\frac{1}{u_e'} + \frac{\partial V_e}{\partial e}ph_e(\theta_w,e)\frac{1}{u_e'} - \frac{dV_e}{d\theta_w}\frac{h_e(\theta_w,e)}{u_e'}. \tag{116}
$$

which is the same as the problem without informality. Notice that in this problem the value functions for entrepreneurs are different from the problem without informality: $V_e(\theta_w) = \mathbb{1}u_w^\varphi + \lambda en^\alpha - \lambda\frac{\beta}{1+\sigma}z^{1+\sigma} - \lambda\frac{\delta}{1+\gamma}n_i^{1+\gamma} - \omega_f(n-n_i) - \lambda u_w - \omega_i n_i$ and after implementation we have $V_e(\theta_e) = \mathbb{1}u_e^\varphi(\theta_e) + \lambda\big(T_n(\cdot)+T_c(\cdot)\big)$. With this definition we can obtain the values of the derivatives:

$$
\begin{aligned}
\frac{\partial V_e}{\partial e} =& \lambda n^\alpha - \lambda\delta n_i^\gamma \frac{\partial n_i}{\partial e} + (\omega_f - \omega_i)\frac{\partial n_i}{\partial e} = \lambda n^\alpha + \frac{\partial n_i}{\partial e}\left[\omega_f - \omega_i - \lambda\delta n_i^\gamma\right], \text{ and} \qquad (117)\\
\frac{\partial V_e}{\partial p} =& -\lambda\beta z^\sigma \frac{\partial z}{\partial p}. \qquad (118)
\end{aligned}
$$

Therefore, replacing these in equation (116), we get:

$$
\begin{aligned}
\underbrace{\frac{d}{d\theta_w}\left[-\lambda\beta z^\sigma \frac{\partial z}{\partial p} p h_e(\theta_w, e)\right]\cdot\frac{1}{u_e'}}_{\text{A}} =& \left[V_w - V_e\right] g(\theta_w, e)\frac{1}{u_e'} + \left(\lambda n^\alpha + \frac{\partial n_i}{\partial e}\left[\omega_f - \omega_i - \lambda\delta n_i^\gamma\right]\right) p h_e(\theta_w, e)\frac{1}{u_e'}\\
&- \underbrace{\frac{dV_e}{d\theta_w}\frac{h_e(\theta_w, e)}{u_e'}}_{\text{B}}. \qquad (119)
\end{aligned}
$$

Recall that z is given implicitly by equation (90a) and it's derivative with respect to p is: $\frac{\partial z}{\partial p} = \frac{z^{1-\sigma(1-\beta z^\sigma)}}{p\beta\sigma}$. Also that $u_e(\theta_e) = \pi - T_c(\pi - z)$, hence: $u_e'(\theta_e) = n^\alpha - T_c'(\cdot)n^\alpha = n^\alpha(1 - T_c'(\cdot)) = n^\alpha(1 - \beta z^\sigma)$. So with this and a little algebra manipulation, we obtain:

$$
\begin{aligned}
\frac{d}{d\theta_w}\left[-\lambda\beta z^\sigma \frac{\partial z}{\partial p} p h_e(\theta_w, e)\right]\cdot\frac{1}{u_e'} =& \frac{d}{d\theta_w}\left[-\lambda\beta z^\sigma \frac{\partial z}{\partial p} p h_e(\theta_w, e)\cdot\frac{1}{u_e'}\right] + \frac{d}{d\theta_w}\left[\frac{1}{u_e'}\right]\cdot\lambda\beta z^\sigma\frac{\partial z}{\partial p} p h_e(\theta_w, e)\\
=& \frac{d}{d\theta_w}\left[-\lambda\frac{z}{\sigma}(1-\beta z^\sigma) h_e(\theta_w, e)\cdot\frac{1}{n^\alpha(1-\beta z^\sigma)}\right] +\\
& \frac{d}{d\theta_w}\left[\frac{1}{n^\alpha(1-\beta z^\sigma)}\right]\cdot\lambda z\frac{(1-\beta z^\sigma)}{\sigma} h_e(\theta_w, e)\\
=& \frac{d}{d\theta_w}\left[-\lambda\frac{z}{n^\alpha\sigma} h_e(\theta_w, e)\right] + \frac{d}{d\theta_w}\left[\frac{1}{n^\alpha(1-\beta z^\sigma)}\right]\cdot\lambda z\frac{(1-\beta z^\sigma)}{\sigma} h_e(\theta_w, e).
\end{aligned}
$$

Notice that the second term in the right can be expressed as:

$$
\begin{aligned}
\frac{d}{d\theta_w}\left[\frac{1}{n^\alpha(1-\beta z^\sigma)}\right]\cdot\lambda z\frac{(1-\beta z^\sigma)}{\sigma} h_e(\theta_w, e) =& -\left(\frac{1}{n^\alpha(1-\beta z^\sigma)}\right)^2\cdot\frac{d\Big(n^\alpha(1-\beta z^\sigma)\Big)}{d\theta_w}\cdot\lambda z\frac{(1-\beta z^\sigma)}{\sigma} h_e(\theta_w, e)\\
=& -\left[\alpha n^{\alpha-1}(1-\beta z^\sigma)\frac{dn}{d\theta_w} - n^\alpha\beta\sigma z^{\sigma-1}\frac{dz}{d\theta_w}\right]\frac{\lambda z}{\sigma n^{2\alpha}(1-\beta z^\sigma)} h_e(\theta_w, e).
\end{aligned}
$$

This implies that part A is:

$$
\begin{aligned}
\frac{d}{d\theta_w}\left[-\lambda\beta z^\sigma \frac{\partial z}{\partial p} p h_e(\theta_w, e)\right]\cdot\frac{1}{u_e'} =& \frac{d}{d\theta_w}\left[-\lambda\frac{z}{n^\alpha\sigma} h_e(\theta_w, e)\right]\\
&- \left[\alpha n^{\alpha-1}(1-\beta z^\sigma)\frac{dn}{d\theta_w} - n^\alpha\beta\sigma z^{\sigma-1}\frac{dz}{d\theta_w}\right]\frac{\lambda z}{\sigma n^{2\alpha}(1-\beta z^\sigma)} h_e(\theta_w, e).
\end{aligned}
$$

For part B, remember the definition of the value function for entrepreneurs after implementation, which means:

$$\begin{aligned}\frac{dV_e}{d\theta_w} =& \varphi \mathbb{1} u_e^{\varphi-1} u_e' \frac{de}{d\theta_w} + \lambda T_n'(\cdot) w_f \left(\frac{dn}{d\theta_w} - \frac{dn_i}{d\theta_w}\right) + \lambda T_c'(\cdot) \left(\frac{d\pi}{d\theta_w} - \frac{dz}{d\theta_w}\right)\\ \frac{dV_e}{d\theta_w} \cdot \frac{h_e(\theta_w,e)}{u_e'} =& \varphi \mathbb{1} u_e^{\varphi-1} p h_e(\theta_w,e) + \lambda T_n'(\cdot) w_f \left(\frac{dn}{d\theta_w} - \frac{dn_i}{d\theta_w}\right) \frac{h_e(\theta_w,e)}{u_e'} + \lambda T_c'(\cdot) \left(\frac{d\pi}{d\theta_w} - \frac{dz}{d\theta_w}\right) \frac{h_e(\theta_w,e)}{u_e'}.\end{aligned}$$

In consequence, equation (119) is:

$$\begin{aligned}\frac{d}{d\theta_w}\left[-\lambda \frac{z}{n^\alpha \sigma} h_e(\theta_w,e)\right] - \left[\alpha n^{\alpha-1}(1-\beta z^\sigma)\frac{dn}{d\theta_w} - n^\alpha \beta \sigma z^{\sigma-1}\frac{dz}{d\theta_w}\right] \frac{\lambda z}{\sigma n^{2\alpha}(1-\beta z^\sigma)} h_e(\theta_w,e)\\ -\left[V_w - V_e\right] g(\theta_w,e)\frac{1}{u_e'} - \left(\lambda n^\alpha + \frac{\partial n_i}{\partial e}\left[\omega_f - \omega_i - \lambda \delta n_i^\gamma\right]\right) p h_e(\theta_w,e) \frac{1}{u_e'}\\ + \left(\varphi \mathbb{1} u_e^{\varphi-1} p h_e(\theta_w,e) + \lambda T_n'(\cdot) w_f \left(\frac{dn}{d\theta_w} - \frac{dn_i}{d\theta_w}\right)\frac{h_e(\theta_w,e)}{u_e'} + \lambda T_c'(\cdot)\left(\frac{d\pi}{d\theta_w} - \frac{dz}{d\theta_w}\right)\frac{h_e(\theta_w,e)}{u_e'}\right) = 0.\end{aligned}$$

Note that there are some simplifications for the previous expression. Doing some algebra we can get that

$$n^\alpha \beta \sigma z^{\sigma-1} \frac{dz}{d\theta_w} \frac{\lambda z h_e(\theta_w,e)}{\sigma n^{2\alpha}(1-\beta z^\sigma)} - \lambda T_c'(\cdot) \frac{dz}{d\theta_w} \frac{h_e(\theta_w,e)}{u_e'} = \lambda T_c'(\cdot) \frac{dz}{d\theta_w} \frac{h_e(\theta_w,e)}{n^\alpha(1-\beta z^\sigma)} - \lambda T_c'(\cdot) \frac{dz}{d\theta_w} \frac{h_e(\theta_w,e)}{n^\alpha(1-\beta z^\sigma)}$$

are the same, so we can eliminate the terms. Alike, we can simplify the following and eliminate some expressions:

$$\begin{aligned}-\alpha n^{\alpha-1}(1-\beta z^\sigma)\frac{dn}{d\theta_w}\frac{\lambda z}{\sigma n^{2\alpha}(1-\beta z^\sigma)} h_e(\theta_w,e) + \lambda T_n'(\cdot) w_f \left(\frac{dn}{d\theta_w} - \frac{dn_i}{d\theta_w}\right)\frac{h_e(\theta_w,e)}{u_e'} =\\ -\frac{\alpha}{n}\frac{dn}{d\theta_w}\frac{\lambda z}{\sigma n^\alpha} h_e(\theta_w,e) + \lambda T_n'(\cdot) w_f \left(\frac{dn}{d\theta_w} - \frac{dn_i}{d\theta_w}\right)\frac{h_e(\theta_w,e)}{u_e'},\end{aligned}$$

Recall that $-\lambda \frac{z}{n^\alpha \sigma} h_e(\theta_w,e)$ is the same as $\lambda \frac{eT_n'(\cdot)}{(1+T_n'(\cdot))(1-T_c'(\cdot))}\left(1 - \frac{\partial n_i}{\partial n}\right) h_e(\theta_w,e)$, thanks to the results found in the previous section. Likewise, $\frac{dn_i}{d\theta_w} = \frac{\partial n_i}{\partial e} p + \frac{\partial n_i}{\partial n}\frac{dn}{d\theta_w}$, therefore we can replace:

$$\begin{aligned}-\frac{\alpha}{n}\frac{dn}{d\theta_w}\frac{\lambda z}{\sigma n^\alpha} h_e(\theta_w,e) + \lambda T_n'(\cdot) w_f \left(\frac{dn}{d\theta_w} - \frac{dn_i}{d\theta_w}\right)\frac{h_e(\theta_w,e)}{u_e'} =\\ -\frac{\alpha}{n}\frac{dn}{d\theta_w}\lambda \frac{eT_n'(\cdot)}{(1+T_n'(\cdot))(1-T_c'(\cdot))}\left(1 - \frac{\partial n_i}{\partial n}\right) h_e(\theta_w,e) + \lambda T_n'(\cdot) w_f \left(\frac{dn}{d\theta_w} - \frac{\partial n_i}{\partial e}p - \frac{\partial n_i}{\partial n}\frac{dn}{d\theta_w}\right)\frac{h_e(\theta_w,e)}{u_e'}.\end{aligned}$$

We can substitute the value of $\alpha e = \frac{w_f(1+T_n'(\cdot))}{n^{\alpha-1}}$ from the FOC of entrepreneurs and also rearranging terms:

$$-\frac{1}{n}\frac{dn}{d\theta_w}\lambda\frac{T_n'(\cdot)}{(1+T_n'(\cdot))(1-T_c'(\cdot))}\left(1-\frac{\partial n_i}{\partial n}\right)\frac{w_f(1+T_n'(\cdot))}{n^{\alpha-1}}h_e(\theta_w,e)+$$
$$\lambda T_n'(\cdot)w_f\left(1-\frac{\partial n_i}{\partial n}\right)\frac{dn}{d\theta_w}\frac{h_e(\theta_w,e)}{n^\alpha(1-\beta z^\sigma)}-\lambda T_n'(\cdot)w_f\frac{\partial n_i}{\partial e}p\frac{h_e(\theta_w,e)}{u_e'}=$$
$$-\frac{dn}{d\theta_w}\lambda\frac{T_n'(\cdot)}{(1-\beta z^\sigma)}\left(1-\frac{\partial n_i}{\partial n}\right)\frac{w_f}{n^\alpha}h_e(\theta_w,e)+$$
$$\lambda T_n'(\cdot)w_f\left(1-\frac{\partial n_i}{\partial n}\right)\frac{dn}{d\theta_w}\frac{h_e(\theta_w,e)}{n^\alpha(1-\beta z^\sigma)}-\lambda T_n'(\cdot)w_f\frac{\partial n_i}{\partial e}p\frac{h_e(\theta_w,e)}{u_e'}=-\lambda T_n'(\cdot)w_f\frac{\partial n_i}{\partial e}p\frac{h_e(\theta_w,e)}{u_e'}.$$

In addition, as $\pi=\theta_e n^\alpha-w_f n-T_n(w_f n)$, thanks to the envelope theorem, we have: $\frac{d\pi}{d\theta_w}=n^\alpha p$, thus:

$$-\lambda n^\alpha p h_e(\theta_w,e)\frac{1}{u_e'}+\varphi\mathbb{1}u_e^{\varphi-1}ph_e(\theta_w,e)+\lambda T_c'(\cdot)\frac{d\pi}{d\theta_w}\frac{h_e(\theta_w,e)}{u_e'}=$$
$$-\lambda n^\alpha p h_e(\theta_w,e)\frac{1}{n^\alpha(1-\beta z^\sigma)}+\varphi\mathbb{1}u_e^{\varphi-1}ph_e(\theta_w,e)+\lambda T_c'(\cdot)n^\alpha p\frac{h_e(\theta_w,e)}{n^\alpha(1-\beta z^\sigma)}=(\varphi\mathbb{1}u_e^{\varphi-1}-\lambda)ph_e(\theta_w,e).$$

With all these modifications the previous equation is:

$$\frac{d}{d\theta_w}\left[-\lambda\frac{z}{n^\alpha\sigma}h_e(\theta_w,e)\right]-\lambda T_n'(\cdot)w_f\frac{\partial n_i}{\partial e}p\frac{h_e(\theta_w,e)}{u_e'}-\big[V_w-V_e\big]g(\theta_w,e)\frac{1}{u_e'}+$$
$$(\varphi\mathbb{1}u_e^{\varphi-1}-\lambda)ph_e(\theta_w,e)-\frac{\partial n_i}{\partial e}\left[\omega_f-\omega_i-\lambda\delta n_i^\gamma\right]ph_e(\theta_w,e)\frac{1}{u_e'}=0,$$

lastly, recall that from the FOC from the entrepreneurs we have: $\omega_f-\omega_i-\lambda\delta n_i^\gamma=-\lambda w_f T_n'(\cdot)$, so the terms: $-\lambda T_n'(\cdot)w_f\frac{\partial n_i}{\partial e}p\frac{h_e(\theta_w,e)}{u_e'}-\frac{\partial n_i}{\partial e}\left[\omega_f-\omega_i-\lambda\delta n_i^\gamma\right]ph_e(\theta_w,e)\frac{1}{u_e'}$ cancel each other. Then,

$$\frac{d}{d\theta_w}\left[-\lambda\frac{z}{n^\alpha\sigma}h_e(\theta_w,e)\right]=\big[V_w-V_e\big]g(\theta_w,e)\frac{1}{u_e'}+(\lambda-\varphi\mathbb{1}u_e^{\varphi-1})ph_e(\theta_w,e),$$

and when we integrate the previous from an specific productivity:

$$\int_{\theta_e}^{\overline{\theta_e}}\frac{d}{d\theta_w}\left[-\lambda\varepsilon_{T_c'}^z\frac{z}{n^\alpha}h_e(\theta_w,s)\right]ds=\int_{\theta_e}^{\overline{\theta_e}}\big[V_w-V_e\big]g(\theta_w,s)\frac{1}{u_e'}ds+\int_{\theta_e}^{\overline{\theta_e}}\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)ph_e(\theta_w,s)ds.$$

Moreover, the left part of the equation is:

$$\int_{\theta_e}^{\overline{\theta_e}}\frac{d}{d\theta_w}\left[-\lambda\varepsilon_{T_c'}^z\frac{z}{n^\alpha}h_e(\theta_w,s)\right]ds=-\lambda\varepsilon_{T_c'}^z\frac{z}{n^\alpha}h_e(\theta_w,\overline{\theta_e})+\lambda\varepsilon_{T_c'}^z\frac{z}{n^\alpha}h_e(\theta_w,e)=\lambda\varepsilon_{T_c'}^z\frac{z}{n^\alpha}h_e(\theta_w,e),$$

as there is 0 marginal taxation at the top of the distribution. Also, recall that $V_w(\theta_w)=\mathbb{1}u_w^\varphi+\lambda T_l(\cdot)$, and $V_e(\theta_e)=\mathbb{1}u_e(\theta_e)^\varphi+\lambda\left(T_n(\cdot)+T_c(\cdot)\right)$, hence: $[V_w-V_e]=\lambda\big[T_l(\cdot)-T_c(\cdot)-T_n(\cdot)\big]$. Finally, we obtain:

$$\lambda\varepsilon^z_{T_c'}\frac{z}{n^\alpha}h_e(\theta_w,e) = \int_{\theta_e}^{\overline{\theta_e}}\lambda\big[T_l(\cdot)-T_c(\cdot)-T_n(\cdot)\big]g(\theta_w,s)\frac{1}{u_e'}ds + \int_{\theta_e}^{\overline{\theta_e}}\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)h_e(\theta_w,s)pds \leftrightarrow$$
$$\int_{\theta_e}^{\overline{\theta_e}}\left(\lambda-\mathbb{1}\varphi u_e^{\varphi-1}\right)h_e(\theta_w,s)pds = \lambda\varepsilon^z_{T_c'}\frac{z}{n^\alpha}h_e - \int_{\theta_e}^{\overline{\theta_e}}\lambda\big[T_l(\cdot)-T_c(\cdot)-T_n(\cdot)\big]g(\theta_w,s)\frac{1}{u_e'}ds, \tag{120}$$

which is the same as the problem without informality.

A.7 Using Bunching to Recover Elasticities

In this section we adapt the method developed by Saez (2010), to recover the model parameters from bunching at kinks and nodges in the tax code.

A.7.1 General Regime vs. Special Regime

Firms with sales below 525.000 soles are eligible to pay approximately 2% of sales instead of 29.5% of profits. For simplicity, we abstract from informality in this section.

Under the general regime, a firm solves,

$$\max \theta n^\alpha - wn - 0.295\left(\theta n^\alpha - wn - z\right) - \beta\frac{z^(1+\sigma)}{1+\sigma} \tag{121}$$

The optimal choices are given by,

$$n^{RG} = \left(\frac{\alpha\theta}{w}\right)^{\frac{1}{1-\alpha}} \qquad z^{RG} = \left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}} \tag{122}$$

Under the RER special regime, the firm problem is as follows,

$$\max \theta n^\alpha - wn - \tau_s\left(\theta n^\alpha - z\right) - \beta\frac{z^(1+\sigma)}{1+\sigma} \tag{123}$$
$$s.t{:}\ \theta n^\alpha - z \leq \overline{y} \tag{124}$$

Unconstrained firms choose,

$$n^{RER} = \left(\frac{(1-\tau_s)\alpha\theta}{w}\right)^{\frac{1}{1-\alpha}} \qquad z^{RER} = \left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}} \tag{125}$$

Let $H(y)$ denote the *observed* distribution of reported sales,

$$H(y) = Prob\left(\theta n^\alpha - z \leq y\right) \tag{126}$$

The functions $H^{RG}(y)$ and $H^{RER}(y)$ are associated with the distributions of income that would prevail if all firms were exposed either to the general regime or the special regime.

$$H^{RG}(y) = Prob\left(\theta n^{\alpha} - z \leq y\right) \tag{127}$$

$$= Prob\left(\theta^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}} \leq y\right) \tag{128}$$

$$= F\left(\left(y + \left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}}\right)^{1-\alpha}\left(\frac{w}{\alpha}\right)^{\alpha}\right) \tag{129}$$

The associated density is,

$$h^{RG}(y) = f\left(\left(y + \left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}}\right)^{1-\alpha}\left(\frac{w}{\alpha}\right)^{\alpha}\right)(1-\alpha)\left(y + \left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}}\right)^{-\alpha}\left(\frac{w}{\alpha}\right)^{\alpha} \tag{130}$$

Similarly,

$$H^{RER}(y) = Prob\left(\theta n^{\alpha} - z \leq y\right) \tag{131}$$

$$= Prob\left(\theta^{\frac{1}{1-\alpha}}\left(\frac{(1-\tau_s)\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}} \leq y\right) \tag{132}$$

$$= F\left(\left(y + \left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}}\right)^{1-\alpha}\left(\frac{w}{(1-\tau_s)\alpha}\right)^{\alpha}\right) \tag{133}$$

and,

$$h^{RER}(y) = f\left(\left(y + \left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}}\right)^{1-\alpha}\left(\frac{w}{(1-\tau_s)\alpha}\right)^{\alpha}\right)(1-\alpha)\left(y + \left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}}\right)^{-\alpha}\left(\frac{w}{(1-\tau_s)\alpha}\right)^{\alpha} \tag{134}$$

Also, tedious but straightforward algebra shows that $h^{RG}(.)$ and $h^{RER}(.)$ are related by,

$$h^{RER}\left((1-\tau_s)^{\frac{\alpha}{1-\alpha}}\left[y + \left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}} - \left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}}\right]\right) \tag{135}$$

$$= h^{RG}(y)\left(\frac{y + \left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}}}{(1-\tau_s)\left[y + \left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}} - \left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}}\right] + \left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}}}\right)^{\alpha} \tag{136}$$

If, when eligible, firms prefer the special regime over the general regime, $h(y) = h^{RER}(y)$ for $y < \overline{y}$. Above the threshold,

$$h(y) = h^{RG}(y) = \tag{137}$$

$$h^{RER}\left((1-\tau_s)^{\frac{\alpha}{1-\alpha}}\left[y+\left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}}-\left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}}\right]\right)\left(\frac{(1-\tau_s)\left[y+\left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}}-\left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}}\right]+\left(\frac{\tau_s}{\beta}\right)^{\frac{1}{\sigma}}}{y+\left(\frac{\tau_\pi}{\beta}\right)^{\frac{1}{\sigma}}}\right)^{\alpha} \tag{138}$$

Name	Expression	Value
ε^{n}	$\frac{\partial n}{\partial(1+T_n'(\cdot))}\frac{1+T_n'(\cdot)}{n}$	$-\frac{1}{1-\alpha}$
ε^{n_i}	$\frac{\partial n_i}{\partial(1+T_n'(\cdot))}\frac{1+T_n'(\cdot)}{n_i}$	$\frac{1}{\gamma}\frac{(1-\beta z^{\sigma})\alpha e n^{\alpha-1}}{\delta n_i^{\gamma}}$
ε^{n}	$\frac{\partial n}{\partial(1-T_c'(\cdot))}\frac{1-T_c'(\cdot)}{n}$	0
ε^{n_i}	$\frac{\partial n_i}{\partial(1-T_c'(\cdot))}\frac{1-T_c'(\cdot)}{n_i}$	$\frac{1}{\gamma}$
ε^{z}	$\frac{\partial z}{\partial(T_c'(\cdot))}\frac{T_c'(\cdot)}{z}$	$\frac{1}{\sigma}$
ε^{l}	$\frac{\partial l}{\partial(1-T_l'(\cdot))}\frac{1-T_l'(\cdot)}{l}$	$\frac{1}{\psi}$
ε^{l_i}	$\frac{\partial l_i}{\partial(1-T_l'(\cdot))}\frac{1-T_l'(\cdot)}{l_i}$	$-\frac{1}{\rho}\frac{\chi l^{\psi}}{\kappa\theta_w^{1+\rho}l_i^{\rho}}$

Table 15: Elasticities